Adaptive *hp*-FEM for Eigenproblems

Model problem:

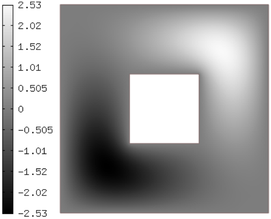
$$-\Delta u + b \cdot \nabla u + cu = \lambda u, \quad u = 0 \text{ on } \partial\Omega$$

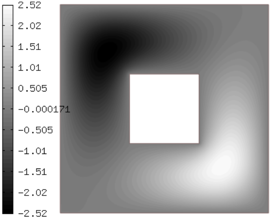
where $b \in \left[L^\infty(\Omega)\right]^2$ and $c \in L^\infty(\Omega)$ is non-negative.

- P. Solin, S. Giani: Adaptive *hp*-FEM for Nonsymmetric Eigenvalue Problems. Computing, accepted.
- P. Solin, S. Giani: An Iterative Finite Element Method for Elliptic Eigenvalue Problems, J. Comp. Appl. Math. 236 (2012) 4582-4599.

Traditional Approach

Call generalized eigensolver after each adaptivity step.





- Ordering of eigenvectors may change between adaptivity steps.
- For repeated eigenvalues, it may return an arbitrary linear combination.
- There is no one optimal mesh to suit all n eigenfunctions.

Novel Iterative Approach

Call generalized eigensolver **only once**:

- Pursue each eigenvalue-eigenvector pair on an individual mesh.
- Adaptive *hp*-FEM combined with Newton or Picard.
- Keeps focus on the selected eigenvalue-eigenvector pair.
- Avoids linear combinations in case of repeated eigenvalues.

Algorithm (Picard)

$$Au^0 = \lambda^0 Bu^0$$

$$\begin{aligned} Au^{m+1} &:= \lambda^m Bu^m \\ \lambda^{m+1} &:= \frac{(u^{m+1})^t Au^{m+1}}{(u^{m+1})^t Bu^{m+1}} \\ m &:= m+1 \end{aligned}$$

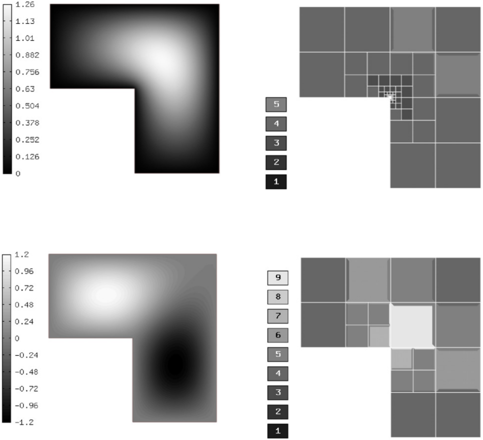
Eigenfunction from coarse mesh used as initial condition on globally refined mesh.
Adaptivity: Based on solution pairs, same as in standard *hp*-FEM.

Algorithm (Newton)

$$0 = f(x, \lambda) := \begin{pmatrix} Ax & - & \lambda\, Bx \\ x^T Bx & - & 1 \end{pmatrix}$$

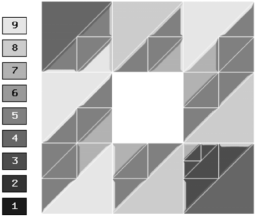
Adaptivity - same as with Picard.

Example 1



Eigenfunctions #1 and #2

Example 2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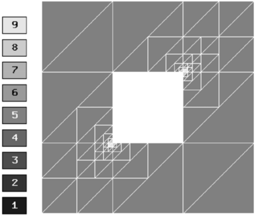
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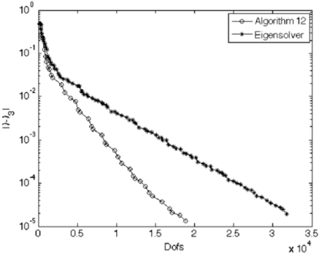
Algorithm 12
Eigensolver
10^{0}
10^{-1}
10^{-2}
10^{-3}
10^{-4}
10^{-5}
$|\lambda-\lambda_3|$
0
0.5
1
1.5
2
2.5
3
3.5
Dofs
$x\ 10^{4}$