Numerical simulation of the electrical activity of the heart

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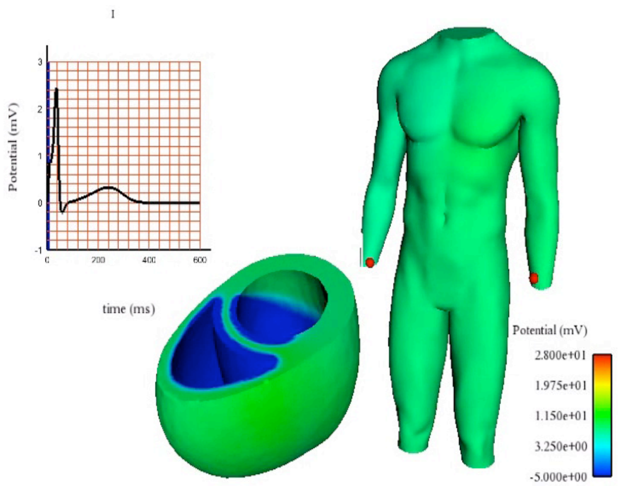
STANFORD
MECHANICAL
ENGINEERING

Heart modelling

CardioSense3D (Project-teams Asclepios, Macs, Reo, Sisyphe)

Heart modelling



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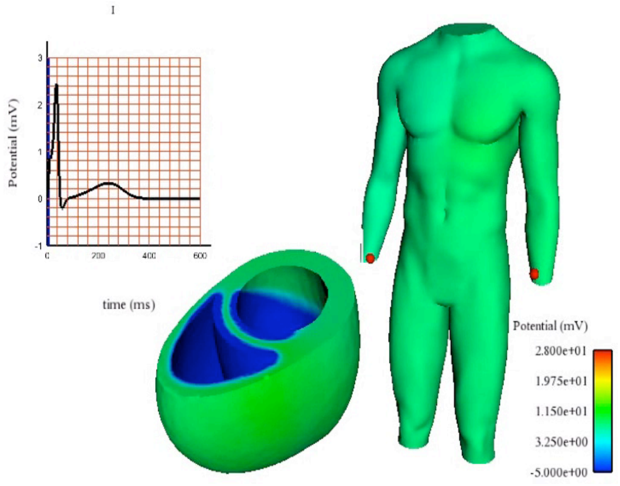

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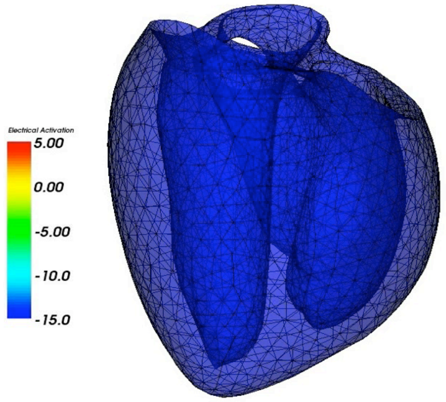

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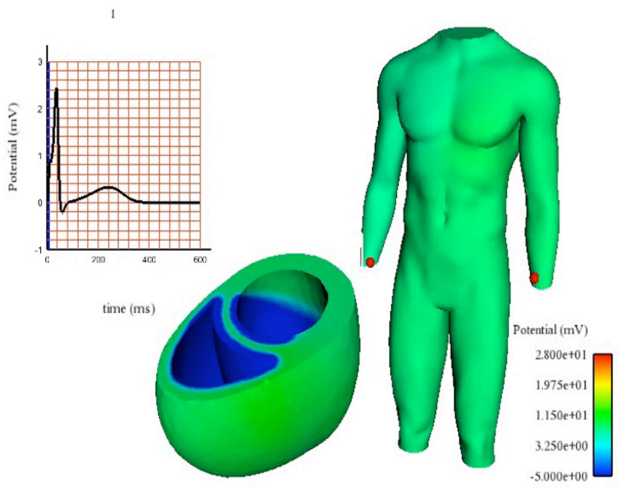

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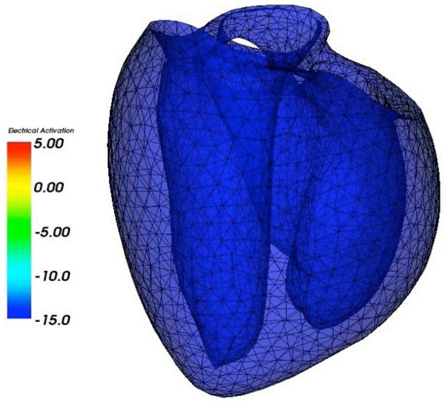

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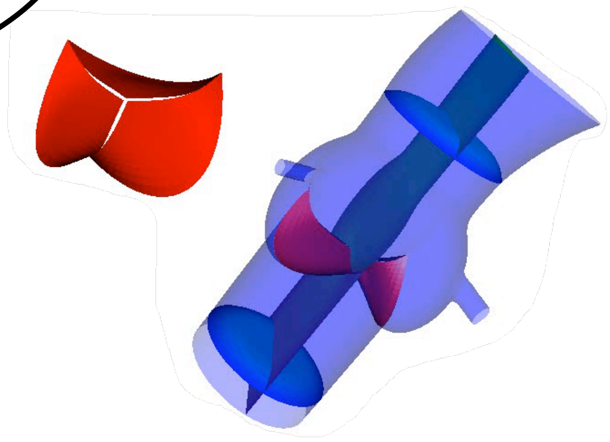

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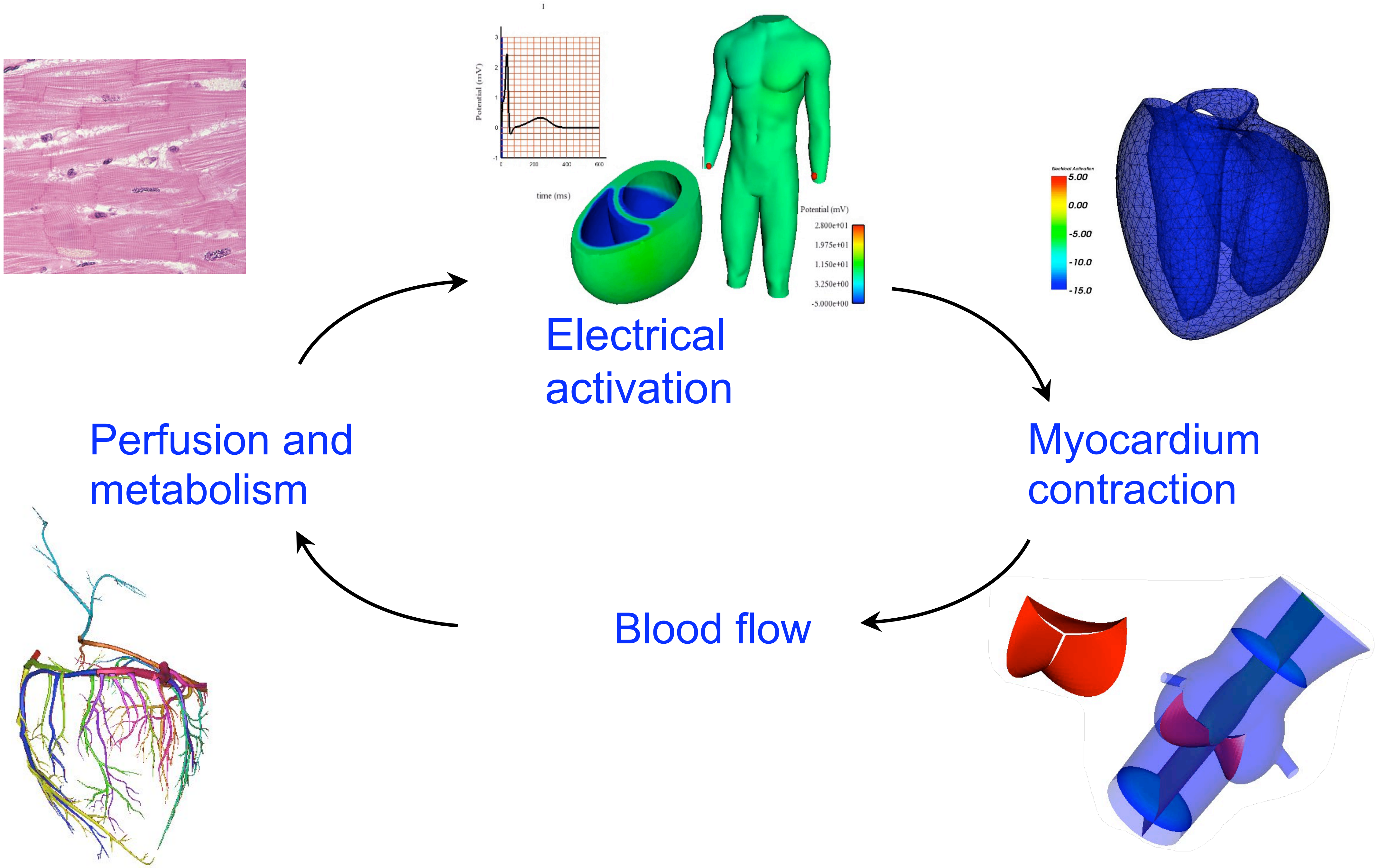

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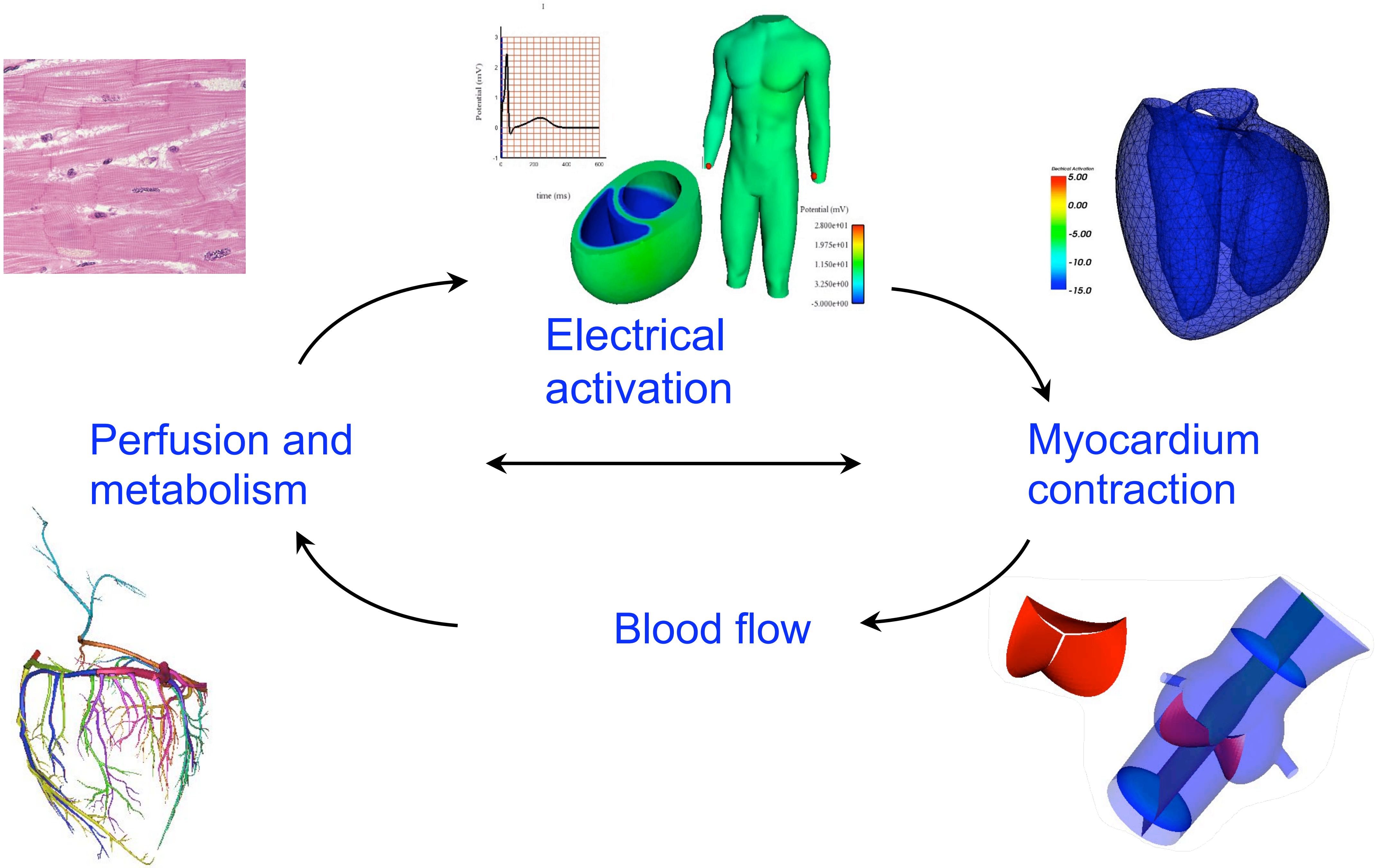

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Electromechanical model

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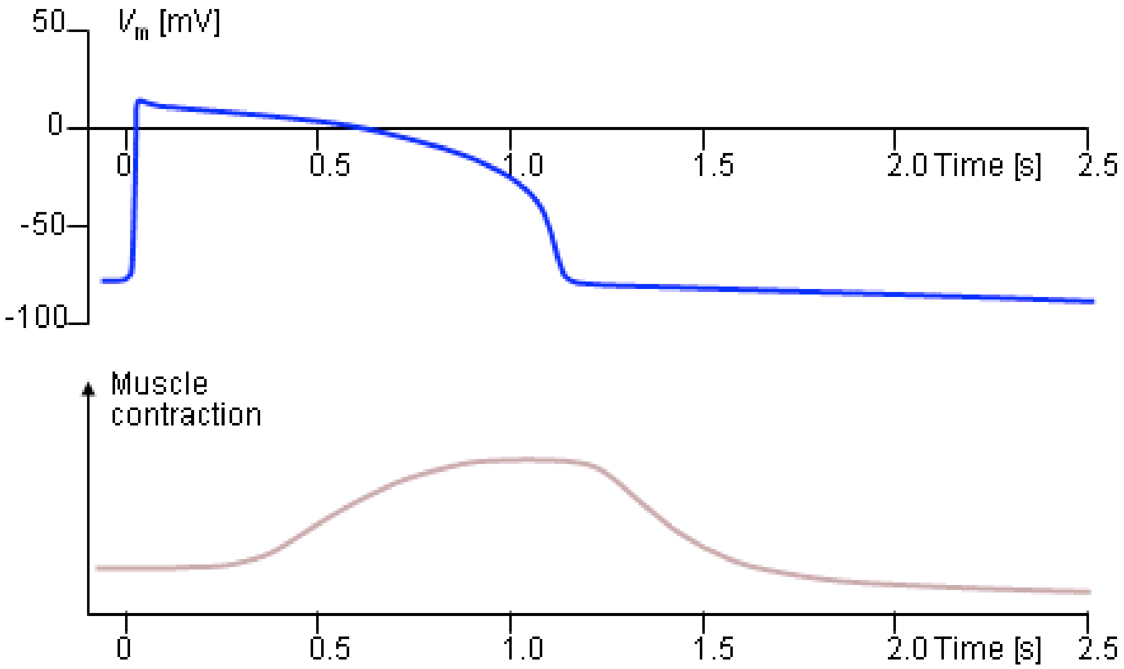

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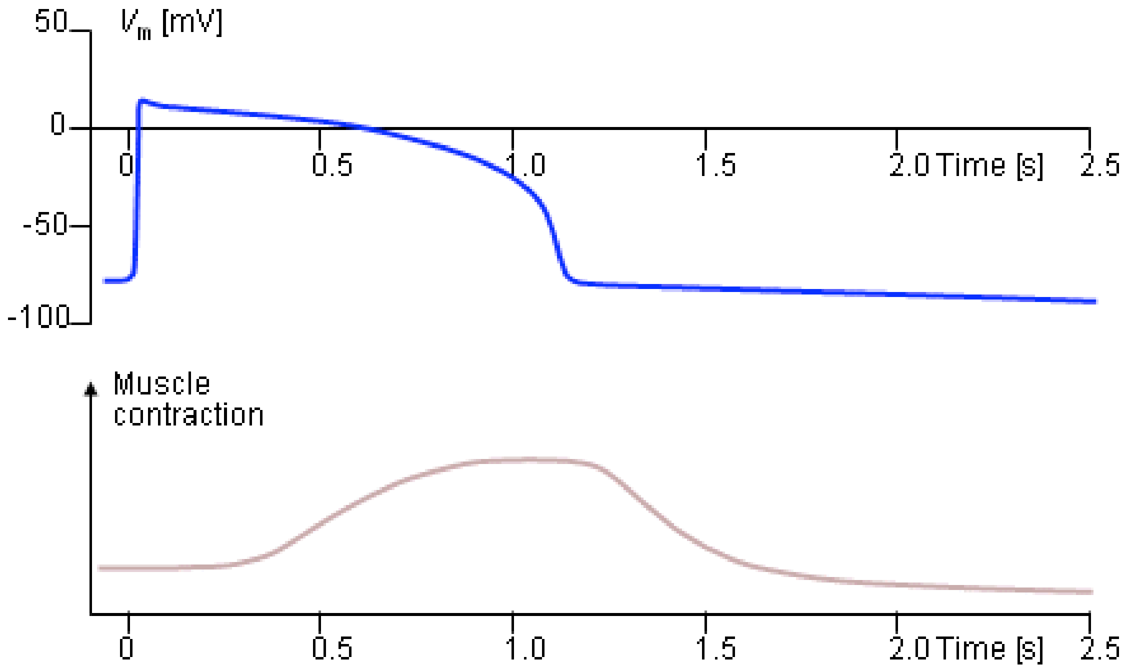

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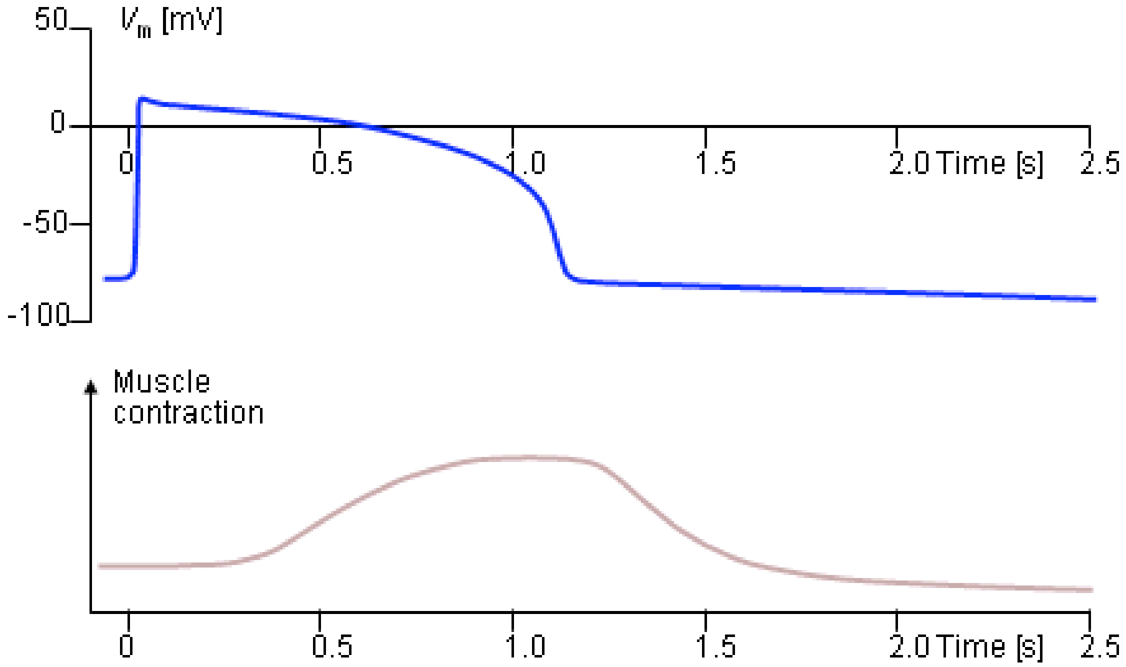

- Total stress in a fiber

“contractile” stress

$$\sigma = \sigma_c + F_P(\varepsilon)$$

strain

Electromechanical model

- **Fiber scale** : active constitutive law

$$\left\{\begin{array}{rcl} \dot{\tau}_c & = & k_c\dot{\varepsilon}_c - (\alpha|\dot{\varepsilon}_c| + |u_c|)\tau_c + \sigma_0|u_c|_+ \\ \dot{k}_c & = & -(\alpha|\dot{\varepsilon}_c| + |u_c|)k_c + k_0|u_c|_+ \\ \sigma_c & = & \tau_c + \nu\dot{\varepsilon}_c \end{array}\right.$$

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- **Tissue scale** :

The fiber model is coupled with a 3D nonlinear material

(D. Chapelle, J. Sainte-Marie, Ph. Moireau)

Myocardium contraction

Chapelle, Moireau, Sainte-Marie, 2007

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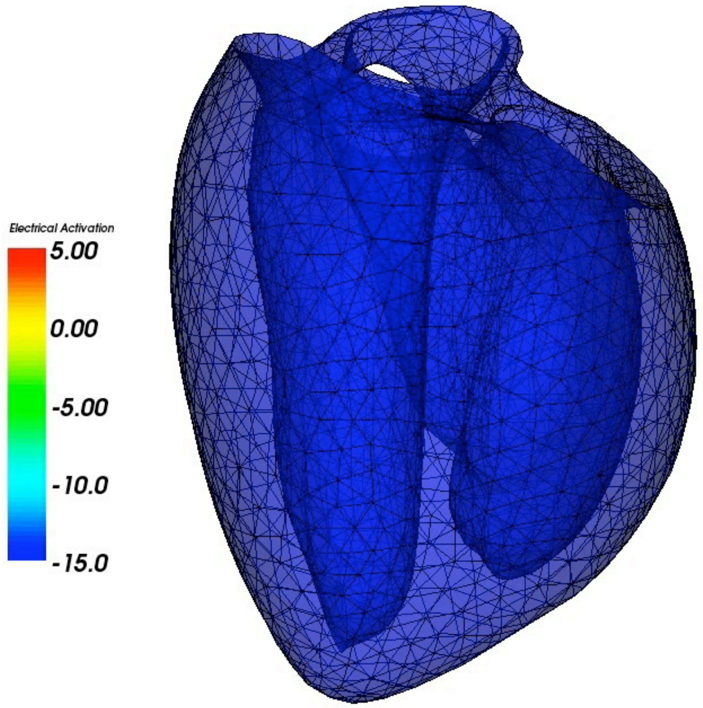

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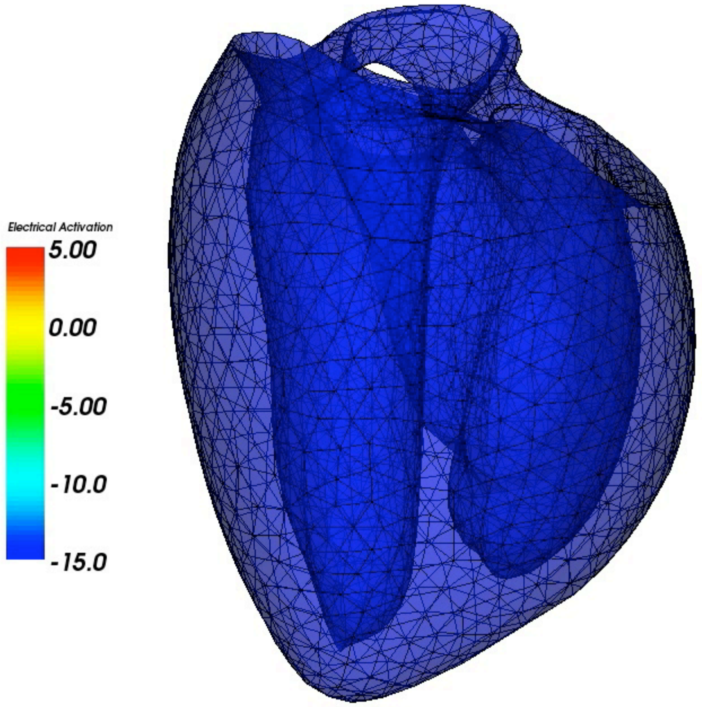

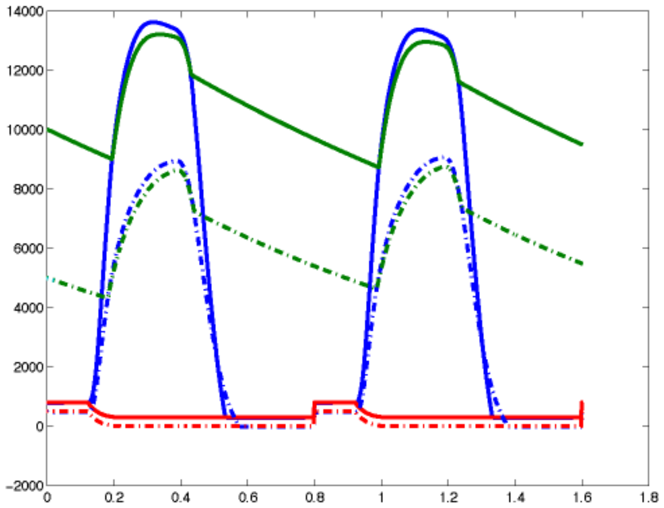

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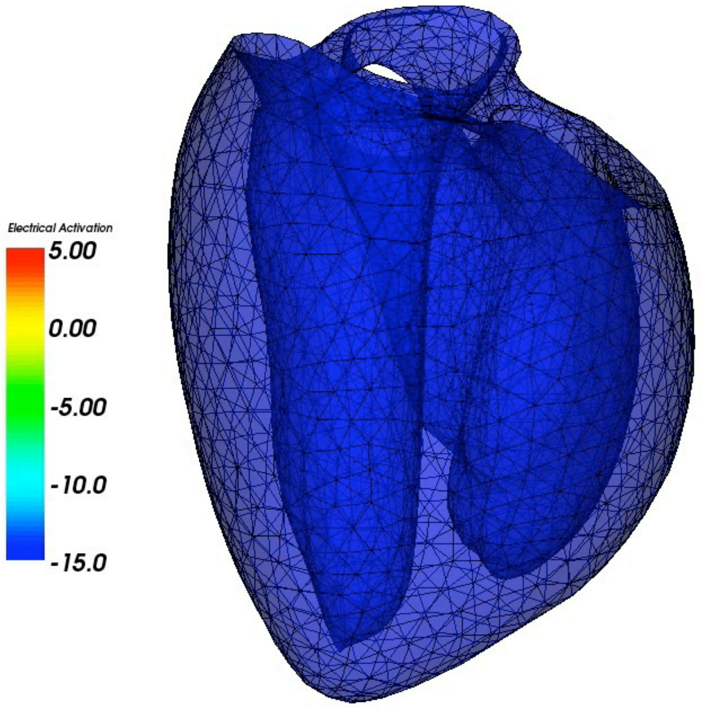

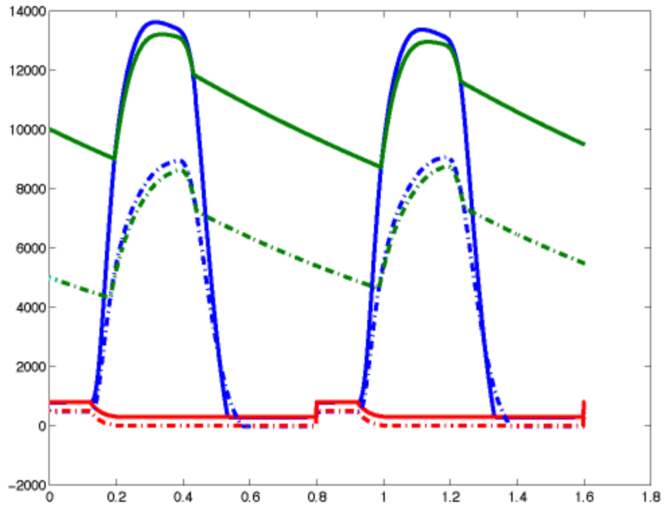

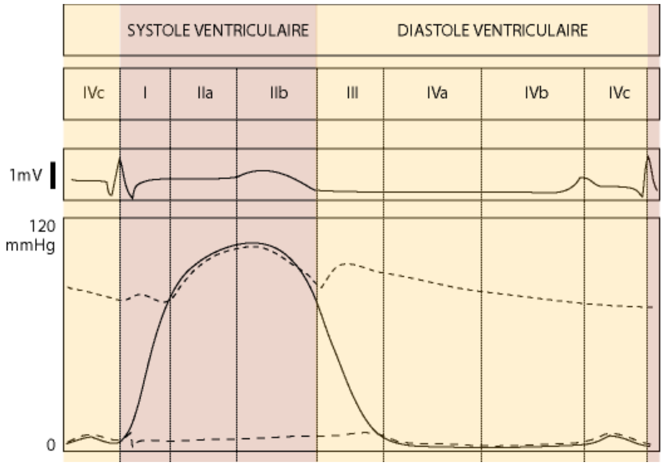

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Outline

- Electrical activity of the heart
  - Electrocardiograms (ECG)
  - Cell scale
  - Tissue scale : the bidomain equations
  - ECG simulation
- Applications
  - Cardiac Resynchronisation Therapy
  - MRI, Magnetohydrodynamics & ECG

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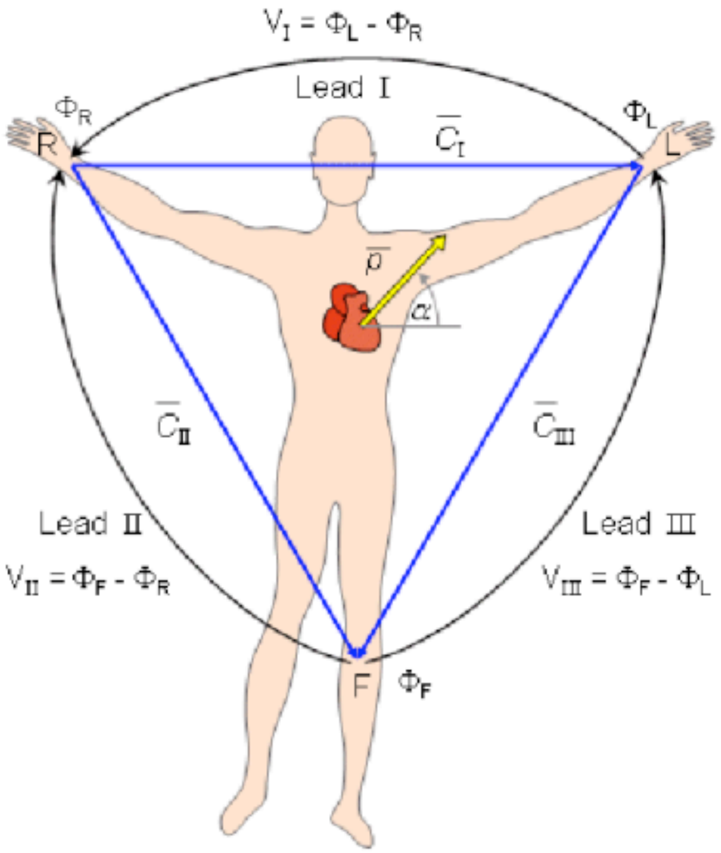
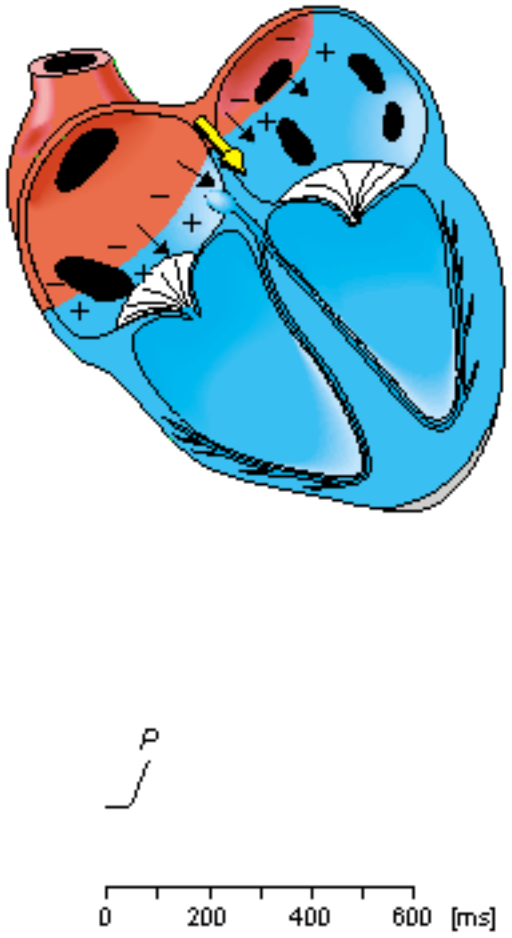
Einthoven assumption
(1895, Nobel prize 1924):

$$\phi_T(M) = \frac{1}{4\pi\epsilon_0}\frac{\boldsymbol{p}\cdot\boldsymbol{OM}}{|\boldsymbol{OM}|^3}$$





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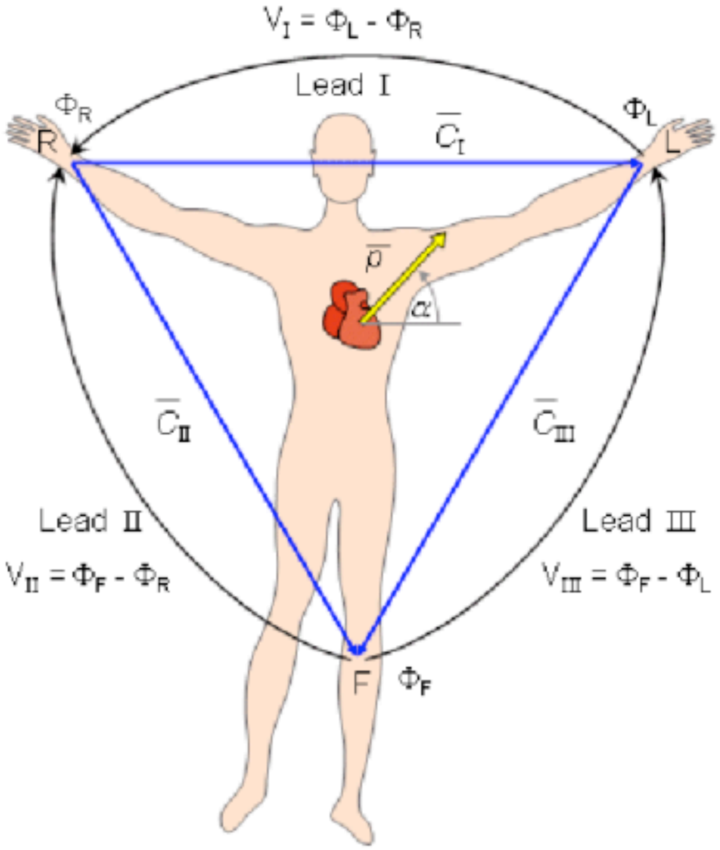

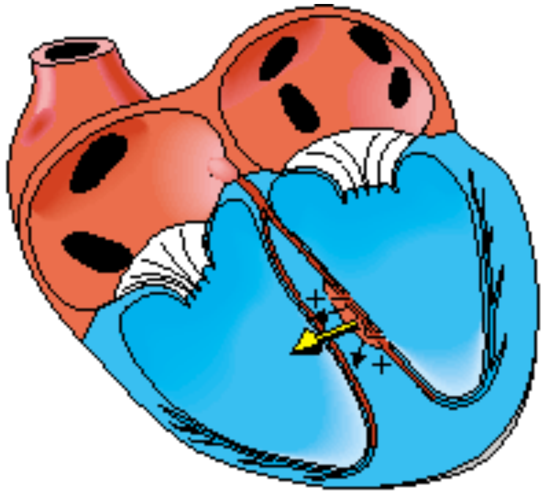

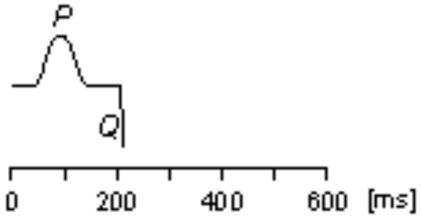

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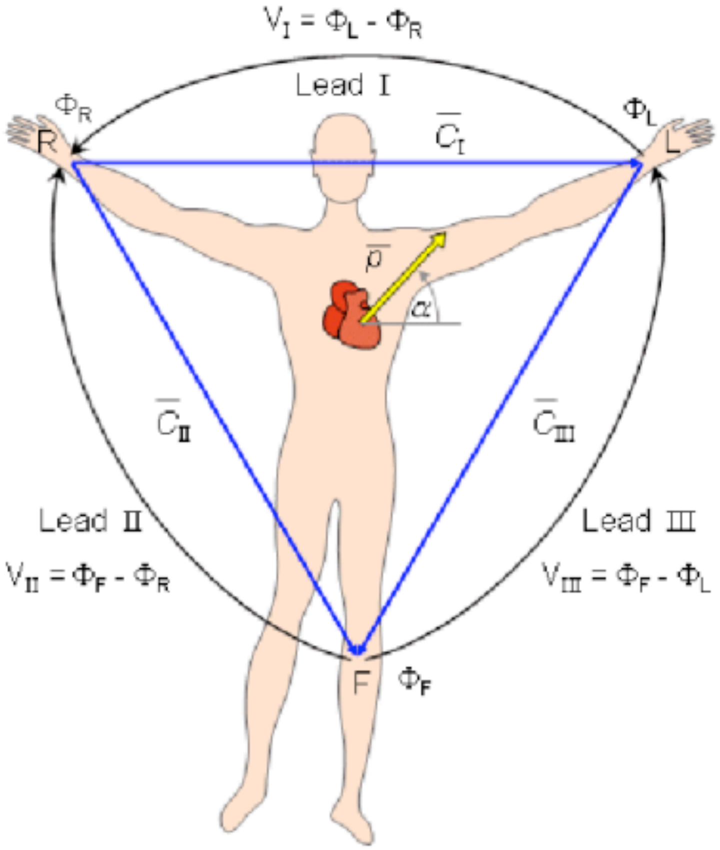

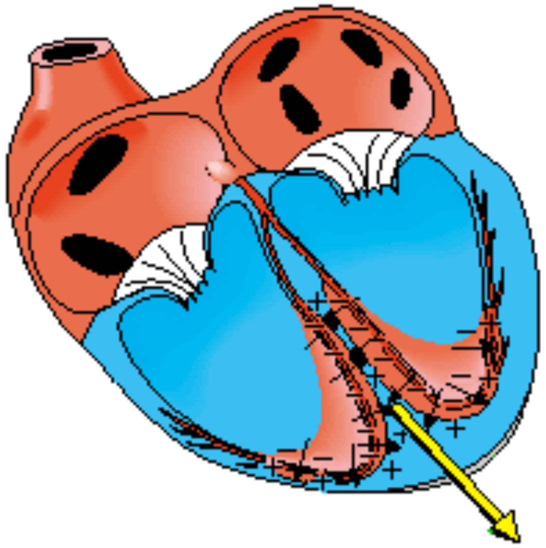

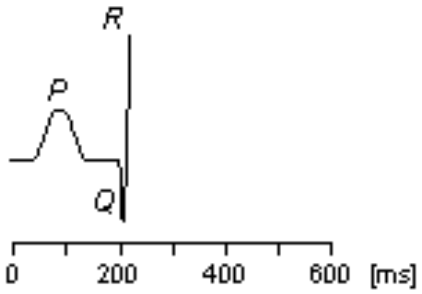

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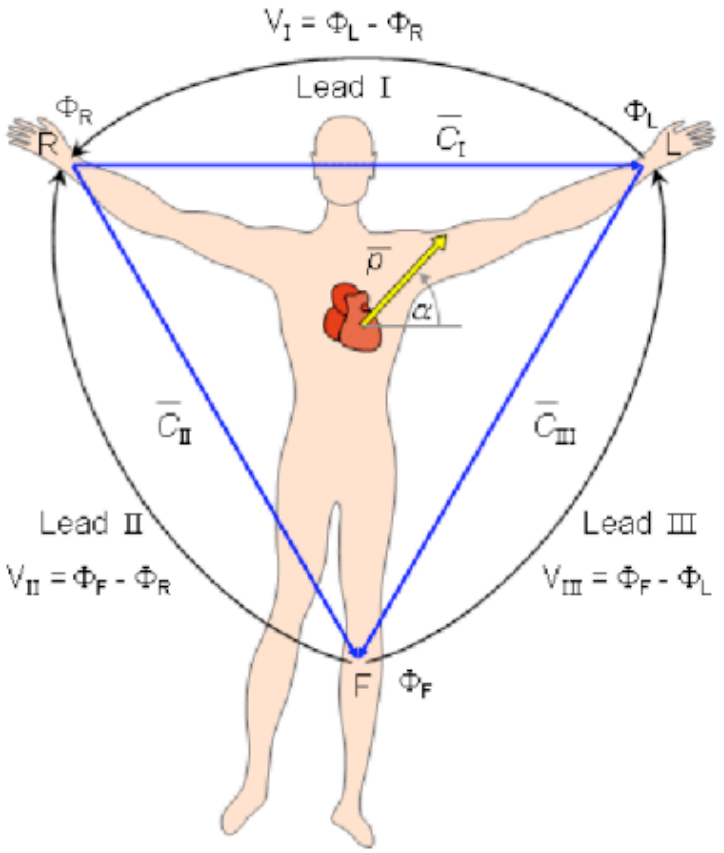

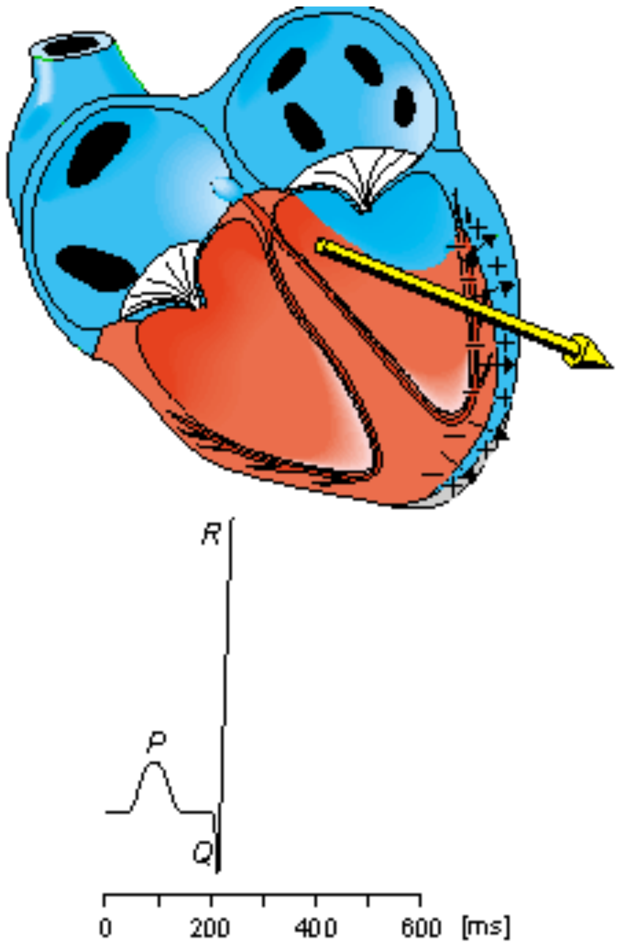

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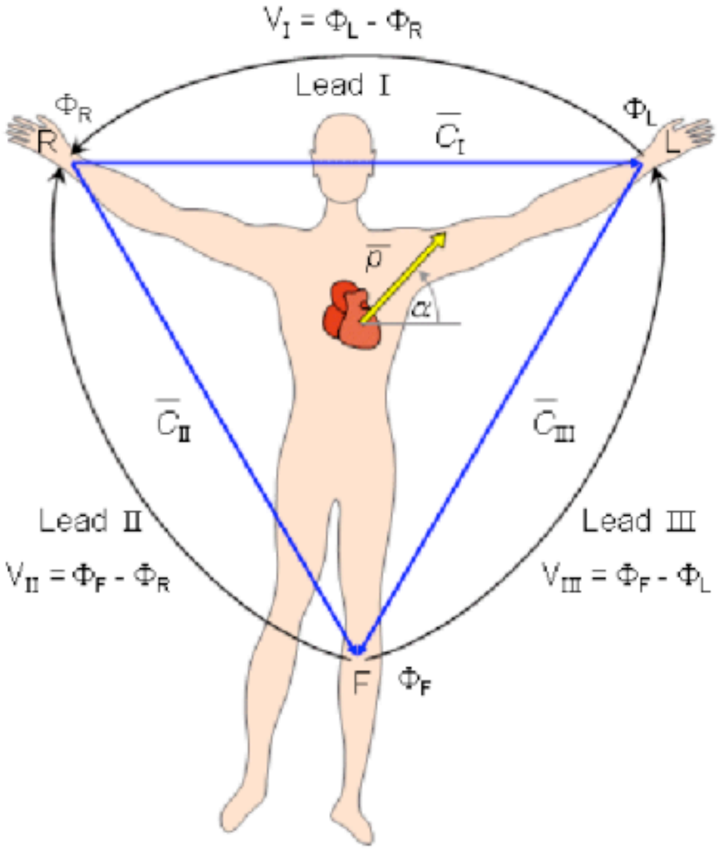

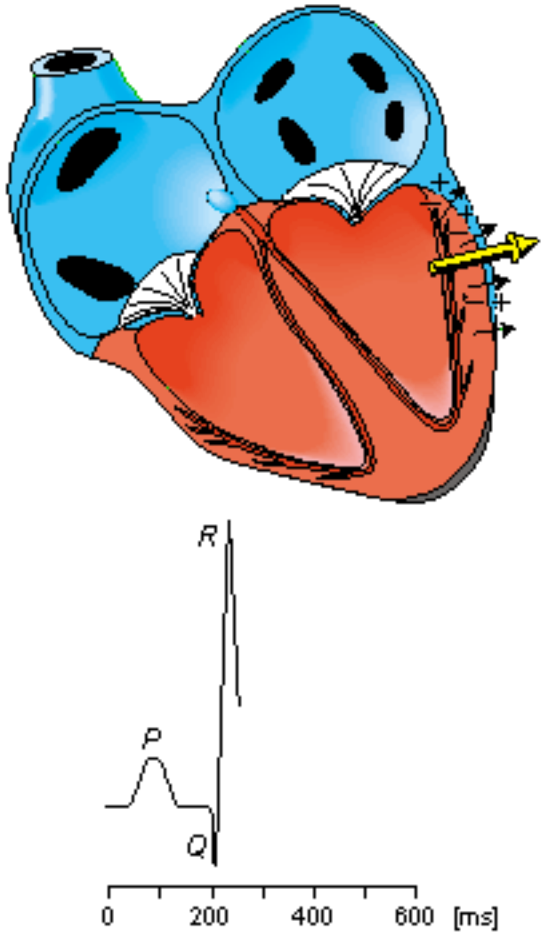

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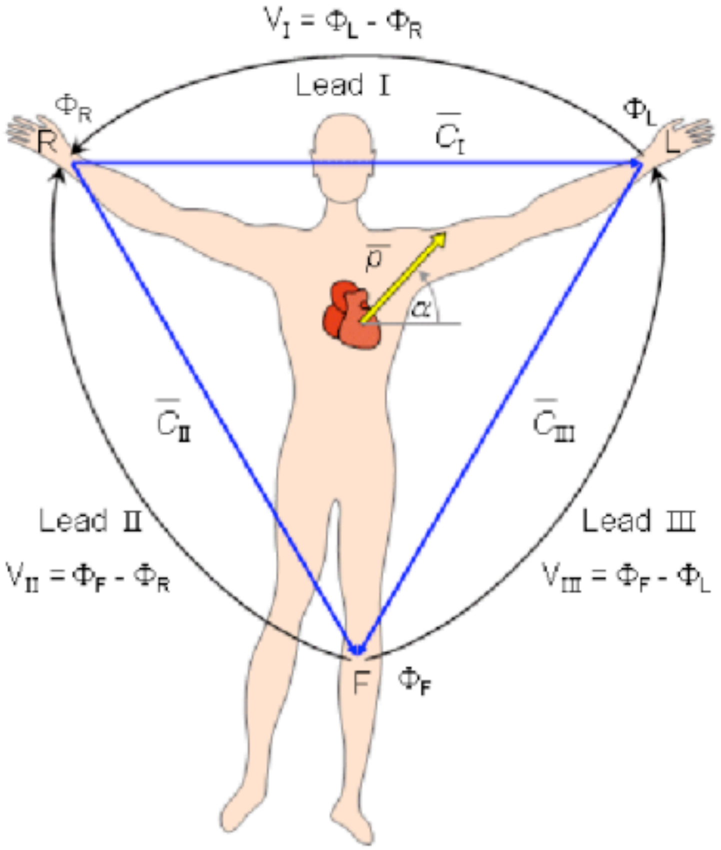

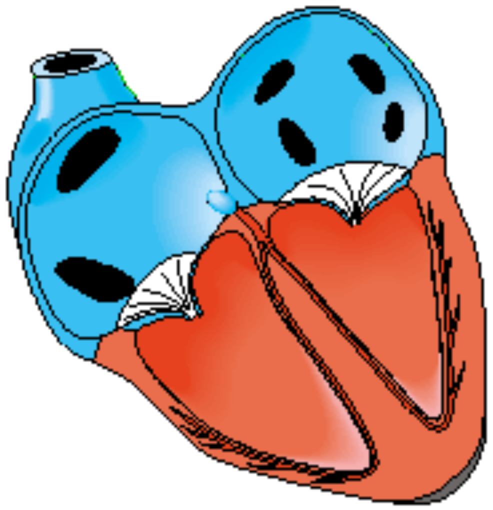

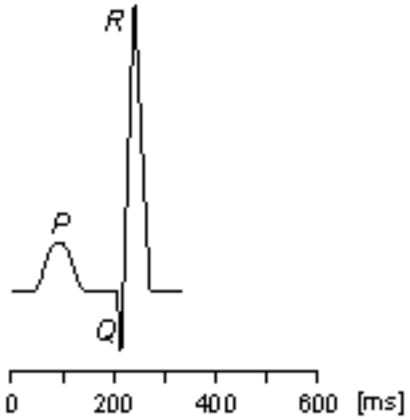

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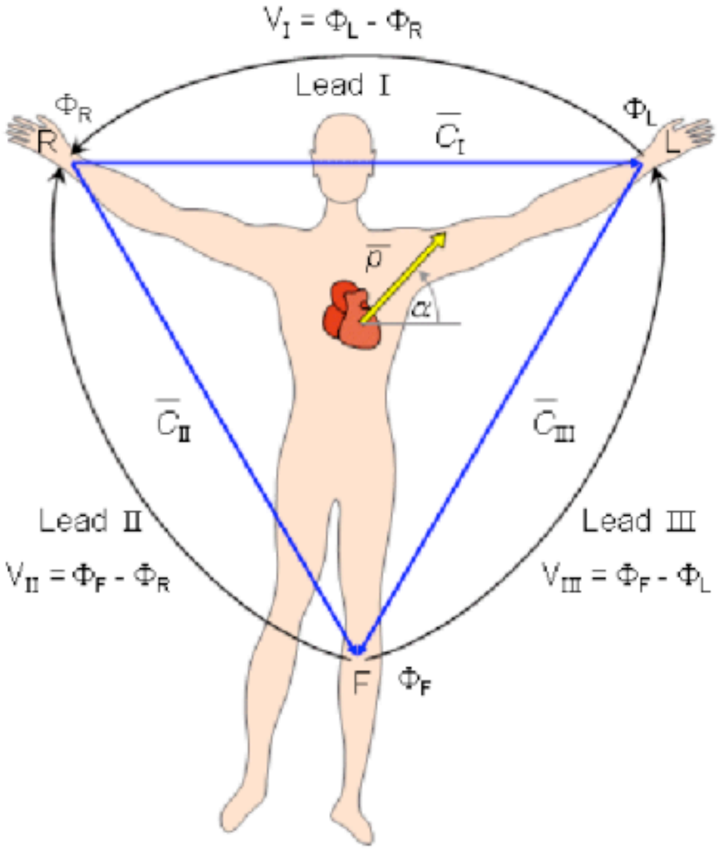

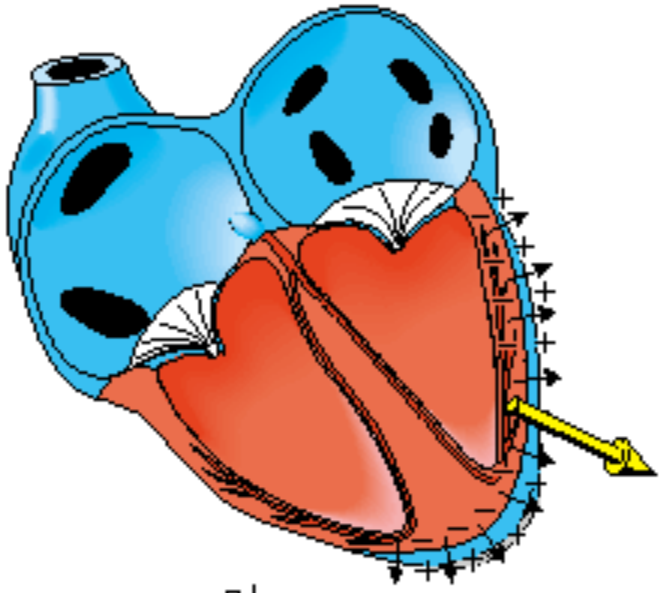

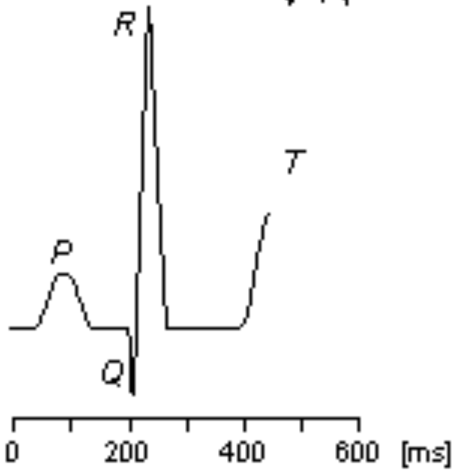

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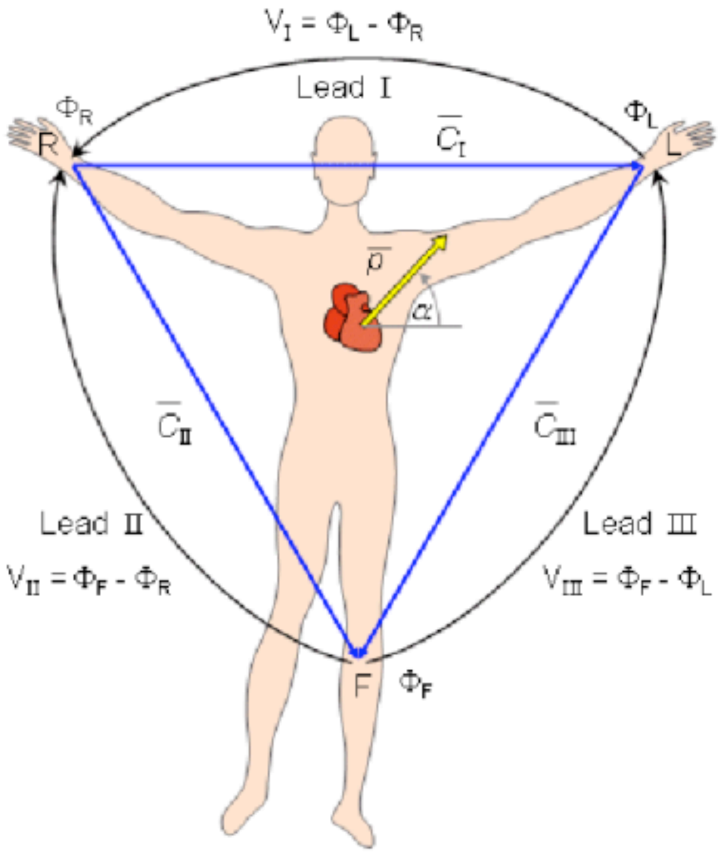

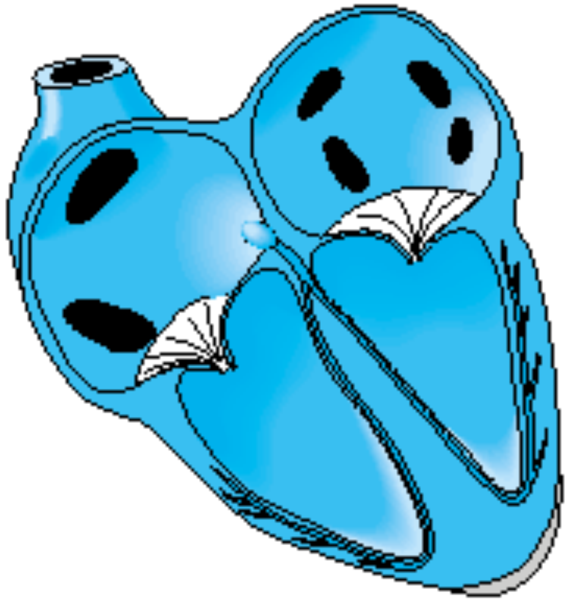

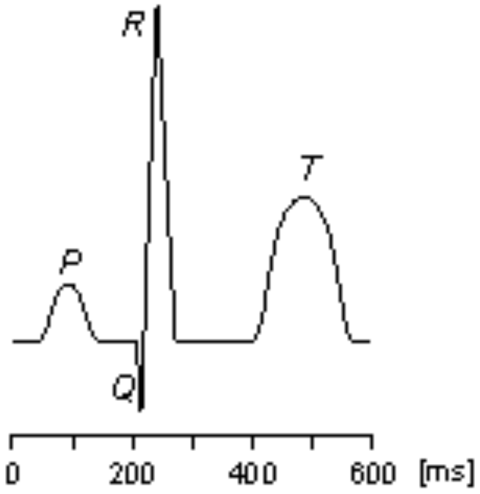

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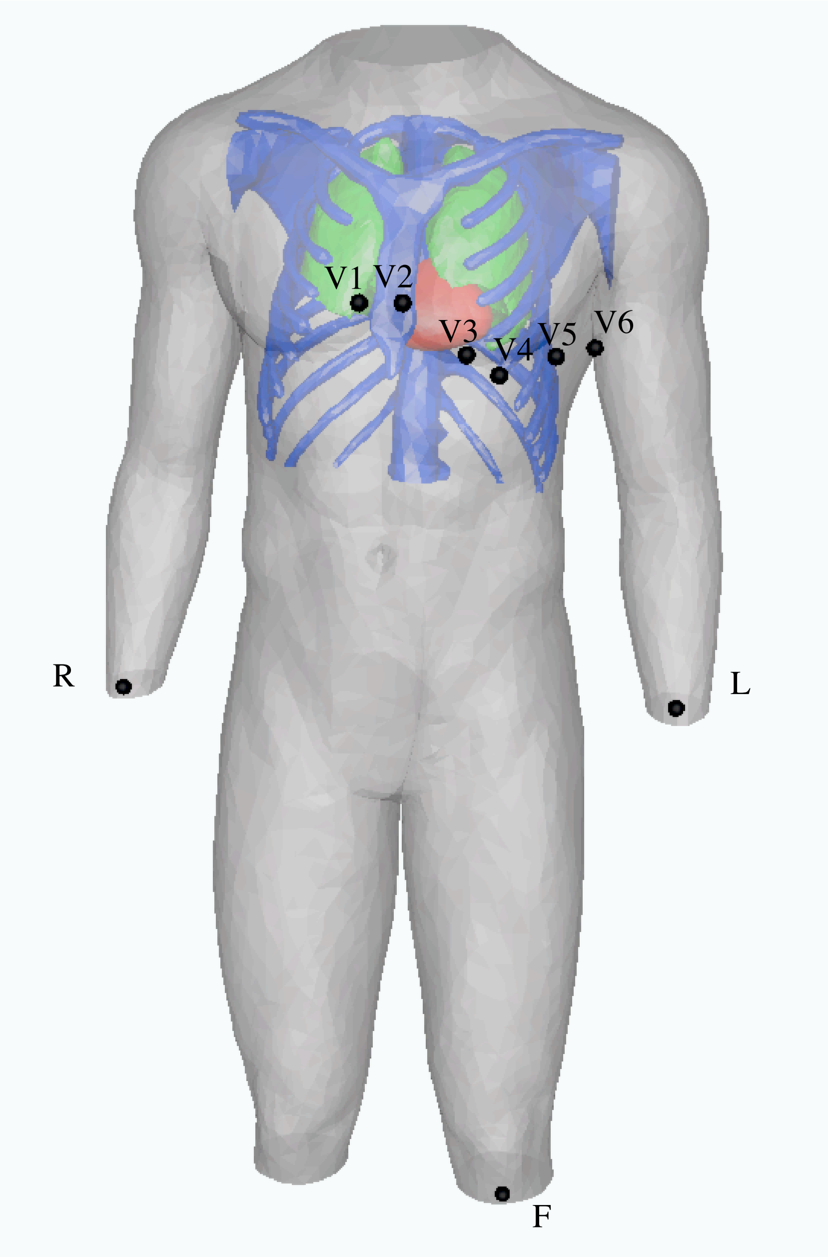
12-lead electrocardiograms

12 measures of electrical potential on the body surface



limb leads (Einthoven)

$$\begin{cases} \mathrm{I} = u_{\mathrm{T}}(L) - u_{\mathrm{T}}(R) \\ \mathrm{II} = u_{\mathrm{T}}(F) - u_{\mathrm{T}}(R) \\ \mathrm{III} = u_{\mathrm{T}}(F) - u_{\mathrm{T}}(L) \end{cases}$$

$$u_W = (u_{\mathrm{T}}(L) + u_{\mathrm{T}}(R) + u_{\mathrm{T}}(F))/3$$

augmented leads

$$\begin{cases} \mathrm{aVR} = \frac{3}{2}(u_{\mathrm{T}}(R) - u_W) \\ \mathrm{aVL} = \frac{3}{2}(u_{\mathrm{T}}(L) - u_W) \\ \mathrm{aVF} = \frac{3}{2}(u_{\mathrm{T}}(F) - u_W) \end{cases}$$

pre-cordial leads

$$\begin{cases} \mathrm{V}_i = u_{\mathrm{T}}(V_i) - u_W \\ i = 1, ..., 6 \end{cases}$$

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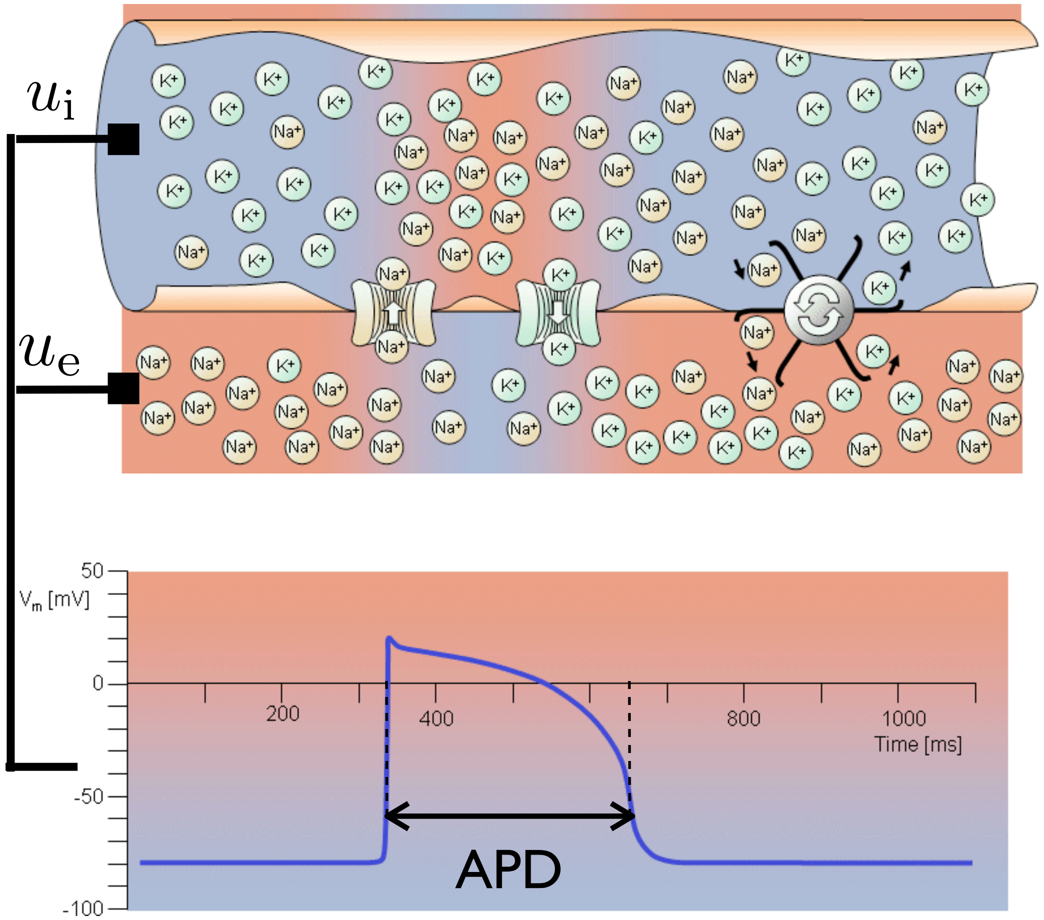
Cell scale



intracellular medium

membrane

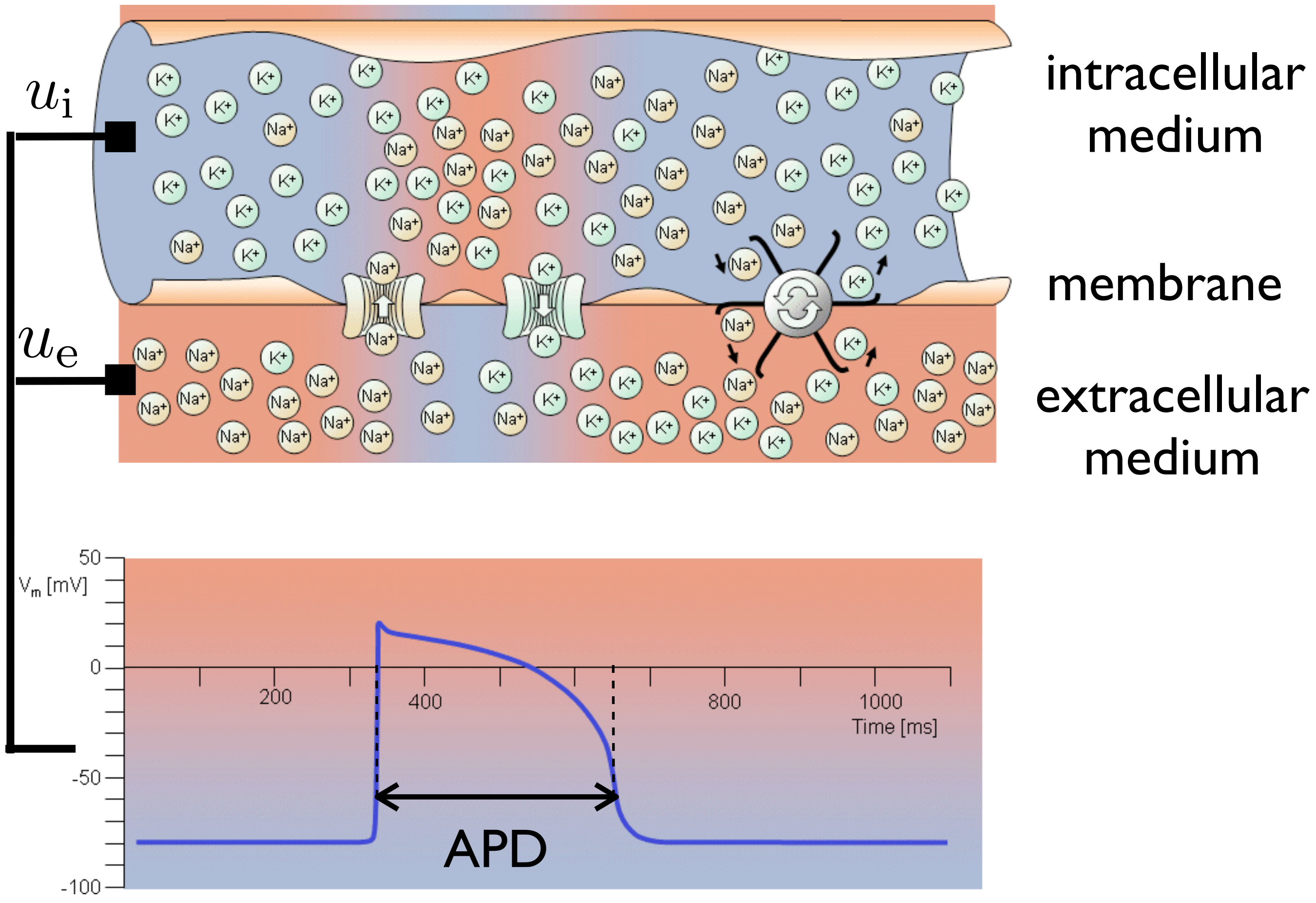
extracellular medium

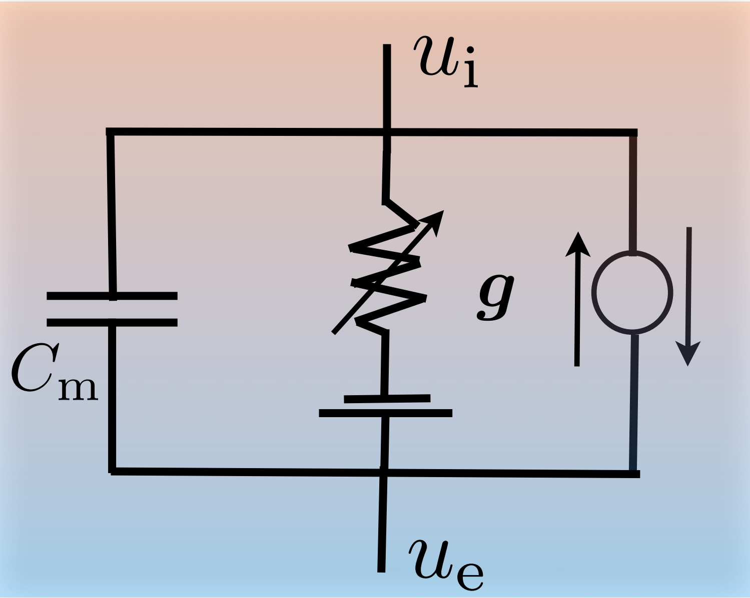
intra-cellular potential u_i

extra-cellular potential u_e

transmembrane potential $V_\mathrm{m} = u_\mathrm{i} - u_\mathrm{e}$

Cell scale





equivalent circuit

intra-cellular potential u_i

extra-cellular potential u_e

transmembrane potential $V_\mathrm{m} = u_\mathrm{i} - u_\mathrm{e}$

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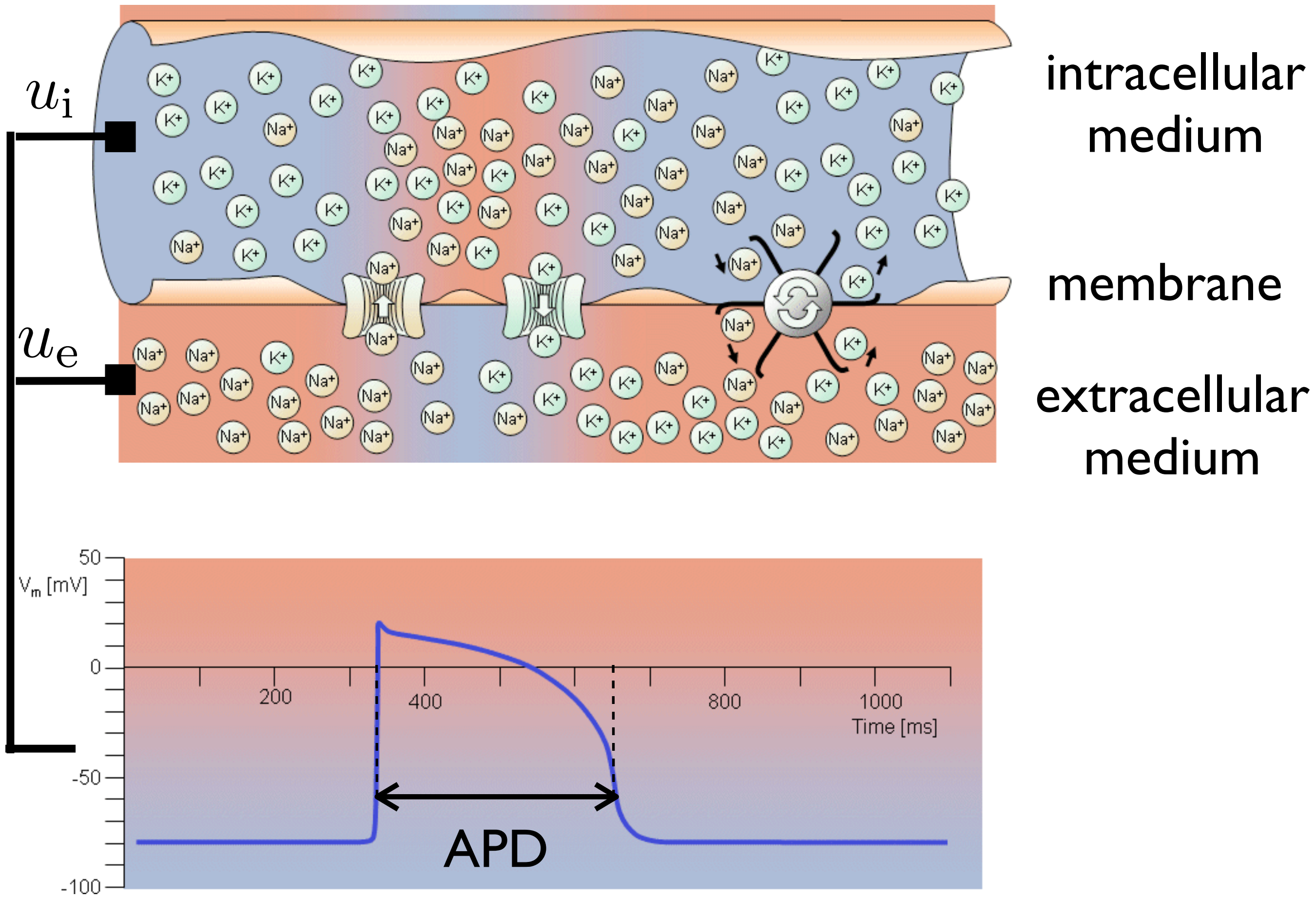
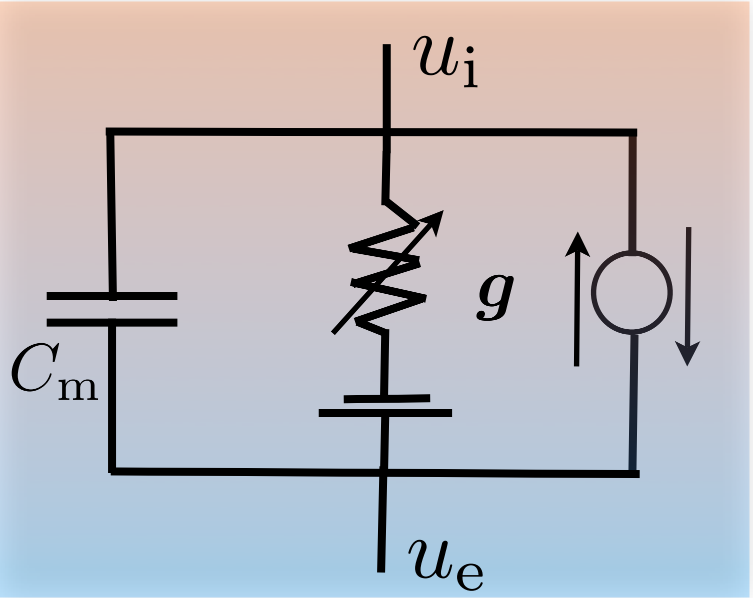




equivalent circuit

intra-cellular potential u_{i}

extra-cellular potential u_{e}

transmembrane potential $V_{\mathrm{m}} = u_{\mathrm{i}} - u_{\mathrm{e}}$

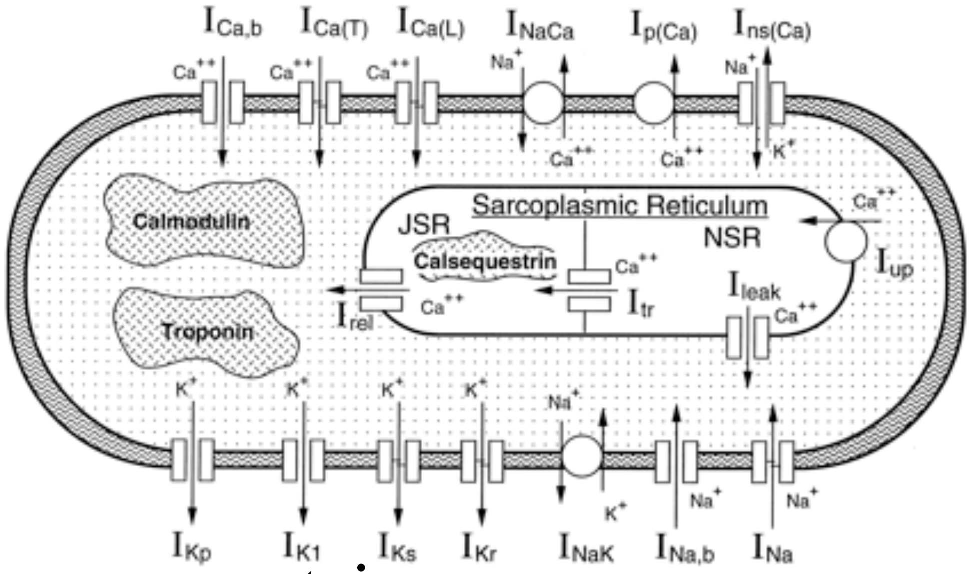
$$\begin{cases} C_{\mathrm{m}} \dfrac{\mathrm{d}V_{\mathrm{m}}}{\mathrm{d}t} + I_{\mathrm{ion}}(V_{\mathrm{m}}, \boldsymbol{g}) = 0 \\ \dfrac{\mathrm{d}\boldsymbol{g}}{\mathrm{d}t} + G(V_{\mathrm{m}}, \boldsymbol{g}) = \mathbf{0} \end{cases}$$

(Hodkin-Huxley 52, Cronin 81, Pullan et al. 05, Sundes et al. 06,...)

Cell scale

Cell scale

- **Physiological models**
    - In *F. Sachse Springer 2004* :
      28 models of cardiac cells !
    - *Noble 60, Luo Rudy 91 & 94, ...*
    - Up to sixty state variables : very difficult to parametrize



Cell scale

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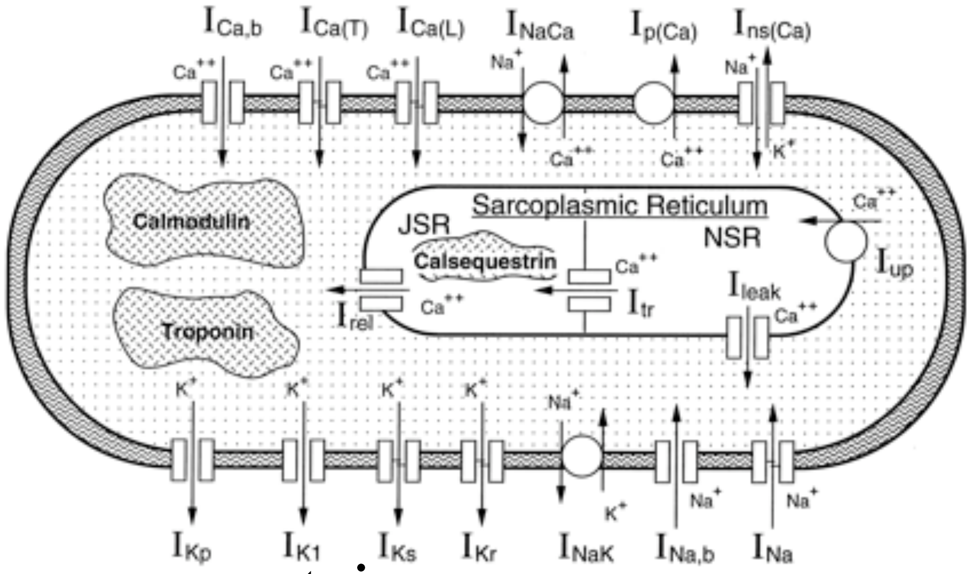

- **Phenomenological models**

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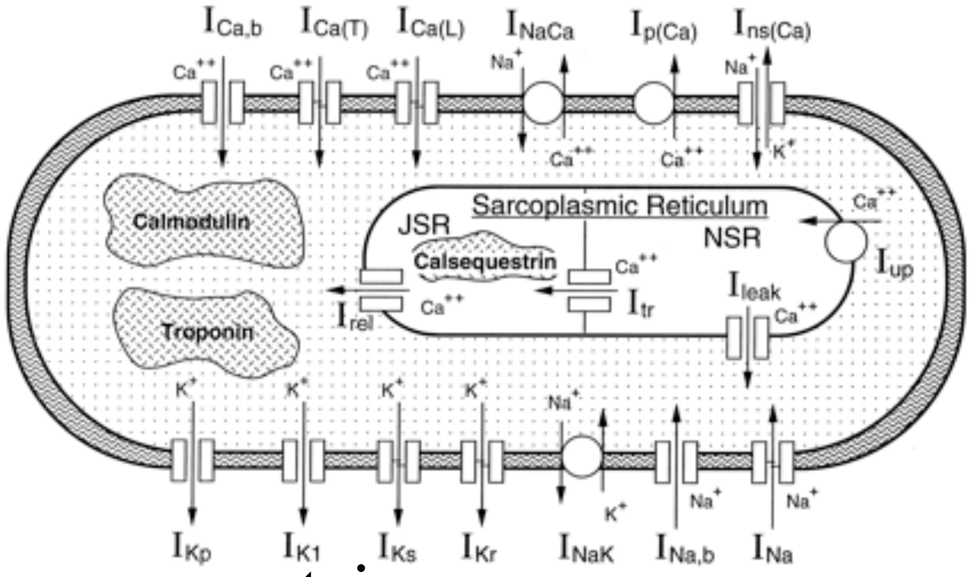

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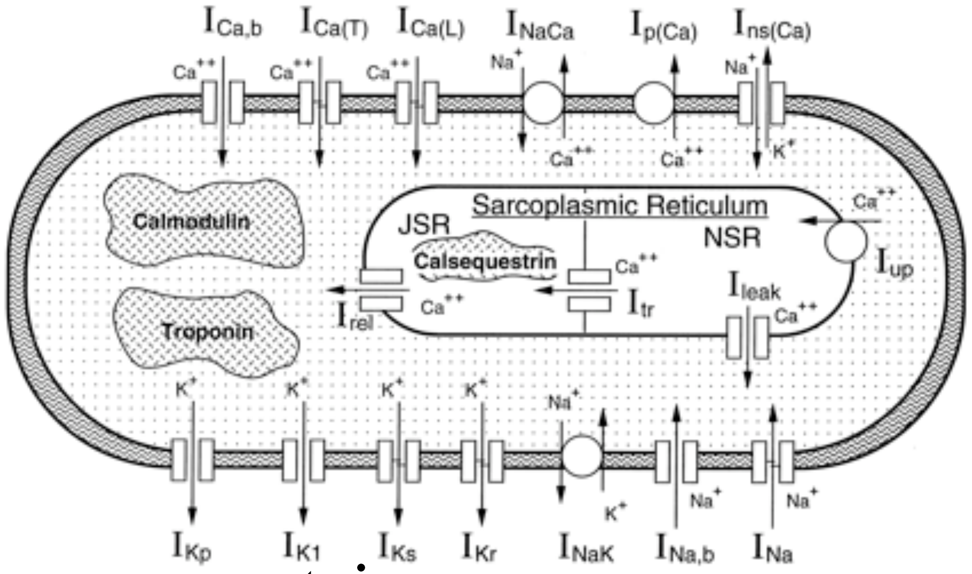

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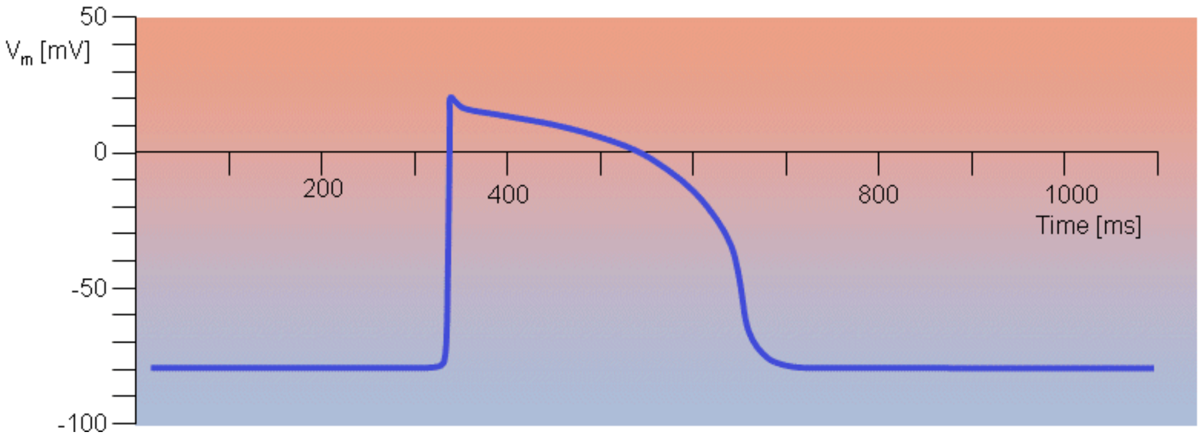

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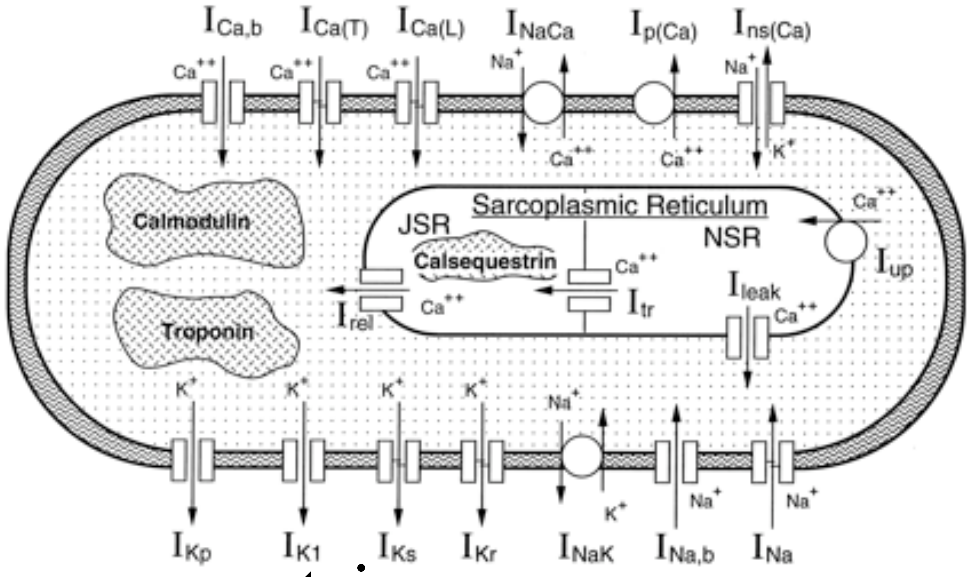

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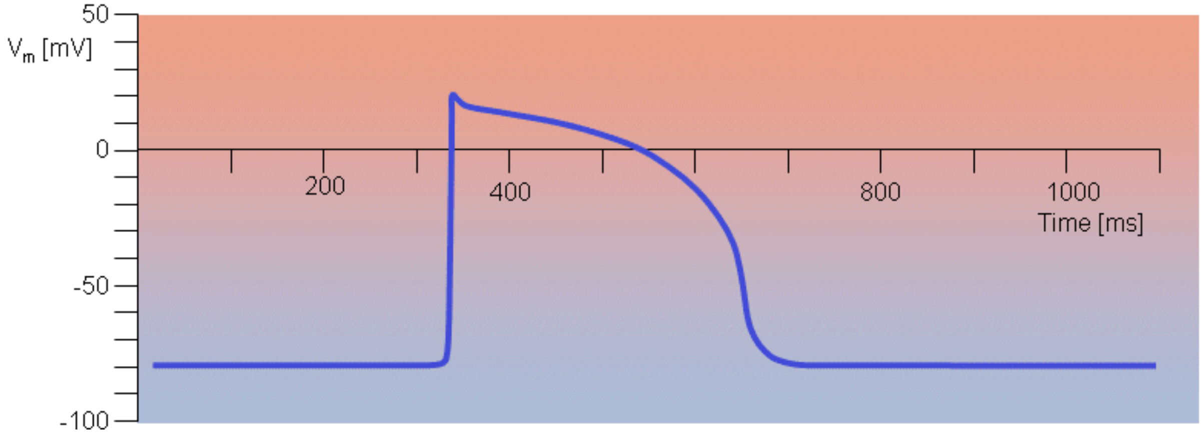

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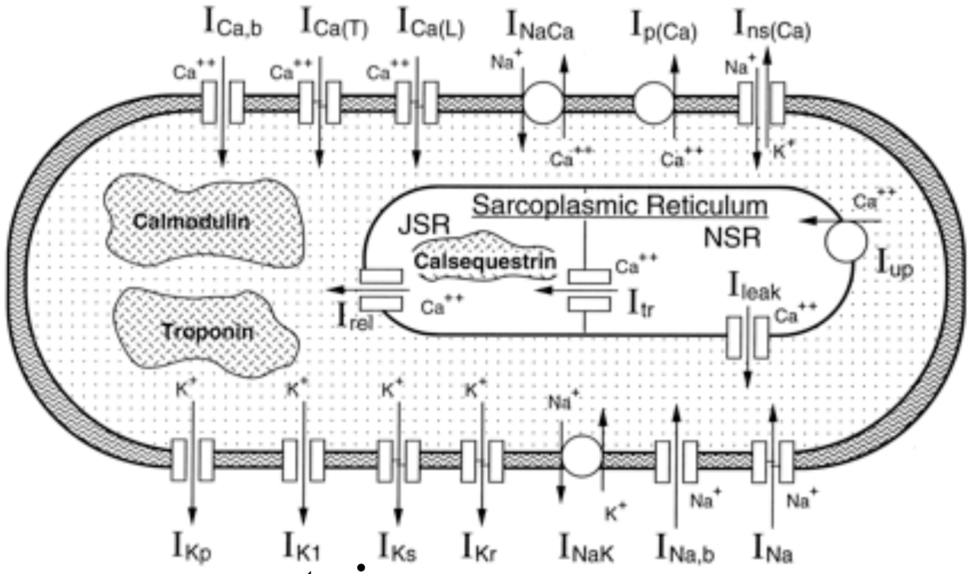

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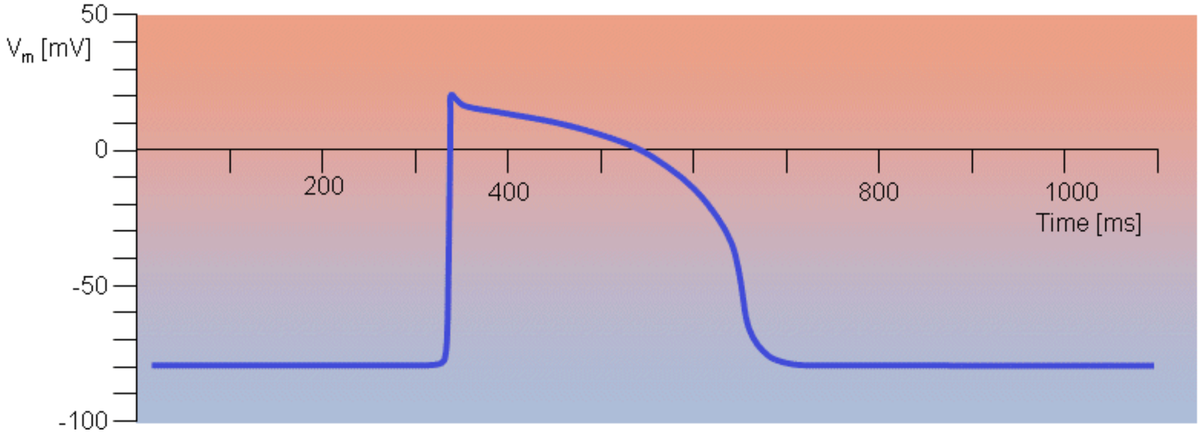

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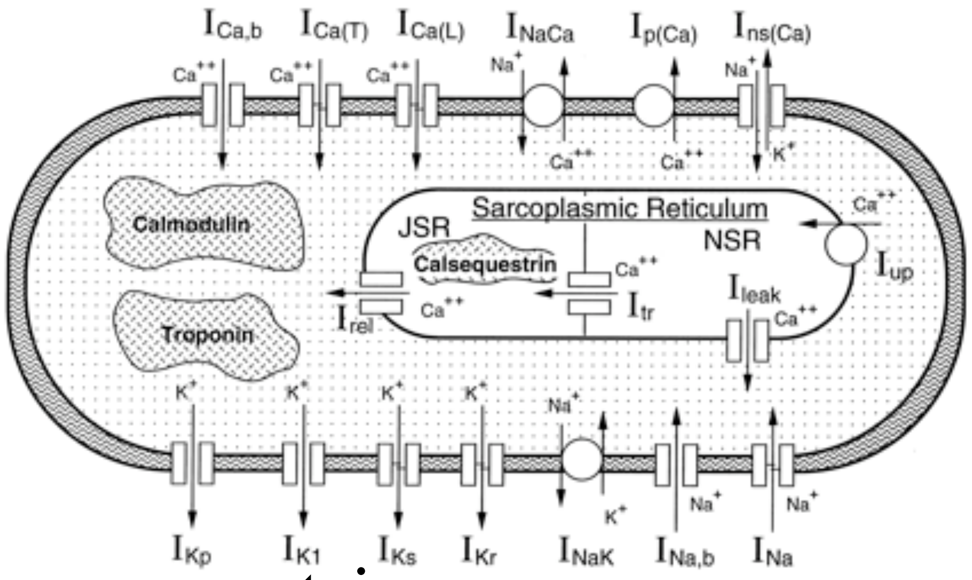

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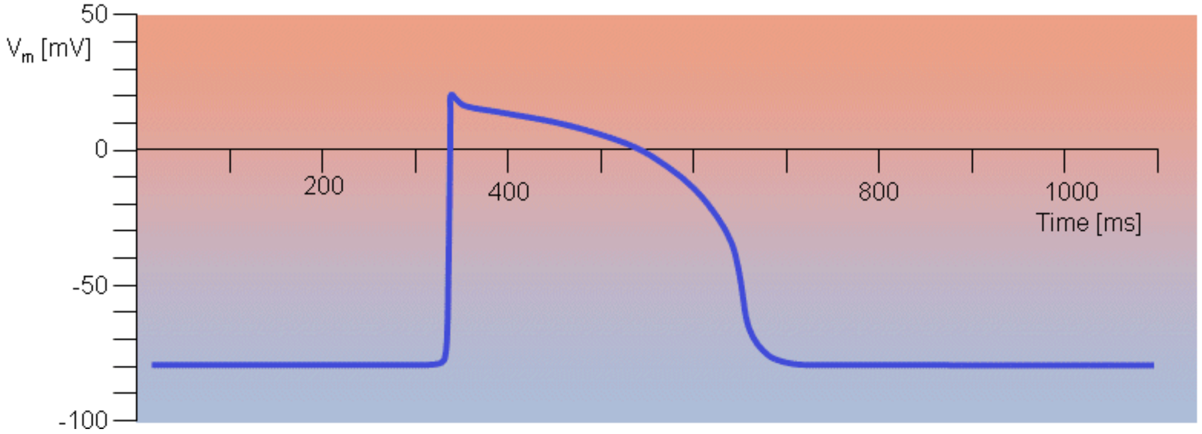

Fitzhugh-Nagumo model

$$\begin{cases} C_{\mathrm{m}}\dfrac{\mathrm{d}V_{\mathrm{m}}}{\mathrm{d}t} + I_{\mathrm{ion}}(V_{\mathrm{m}},\boldsymbol{g}) = 0 \\ \dfrac{\mathrm{d}\boldsymbol{g}}{\mathrm{d}t} + G(V_{\mathrm{m}},\boldsymbol{g}) = \mathbf{0} \end{cases}$$

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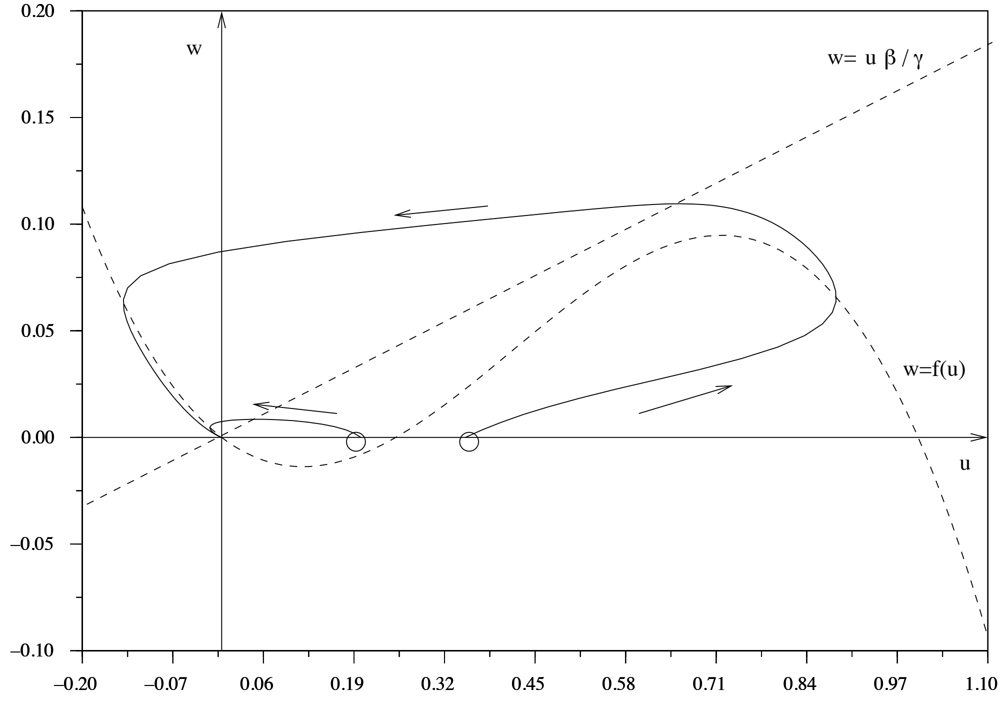

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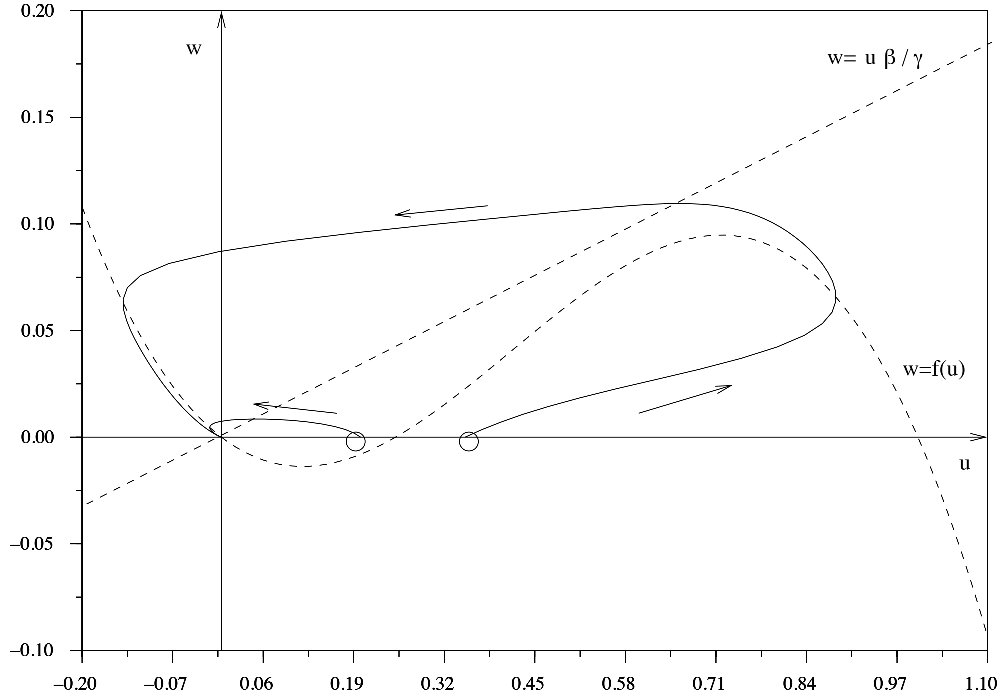

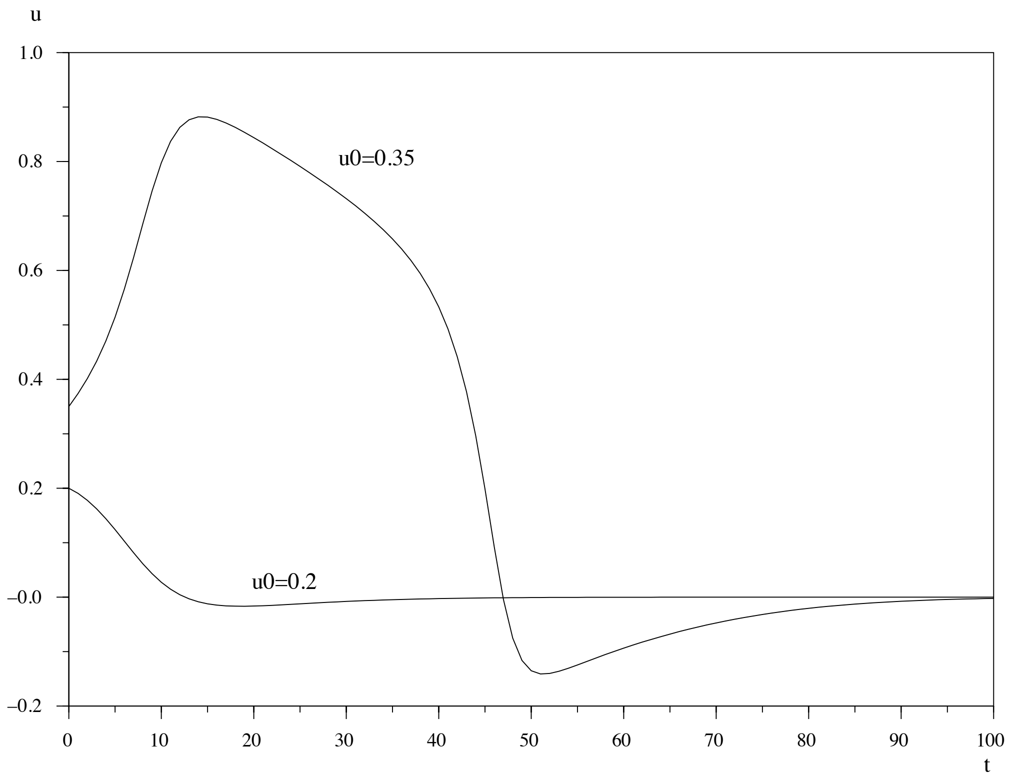

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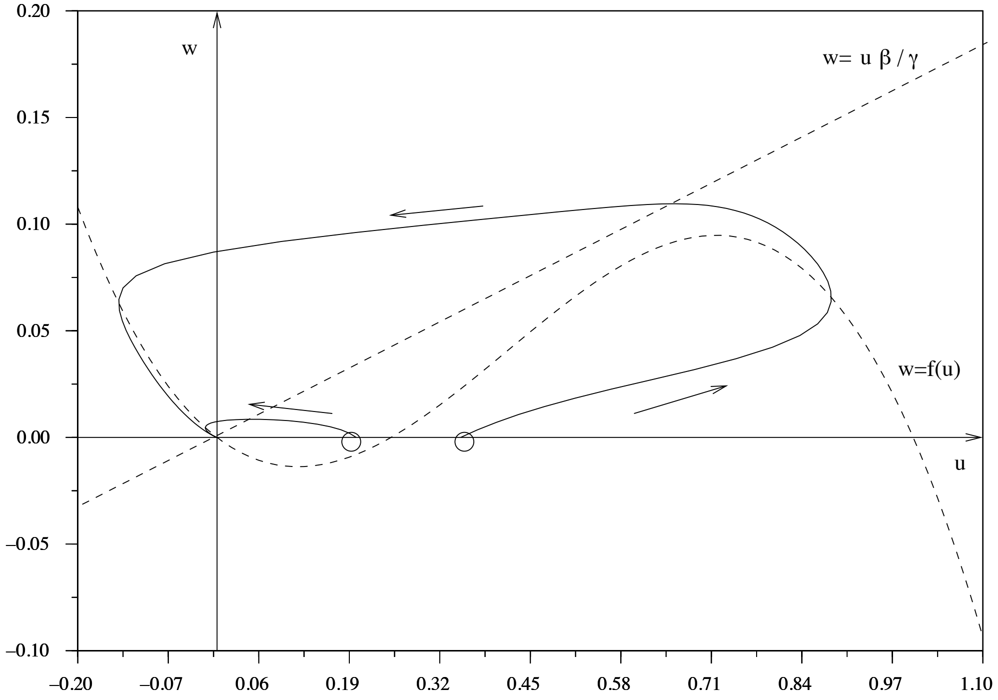

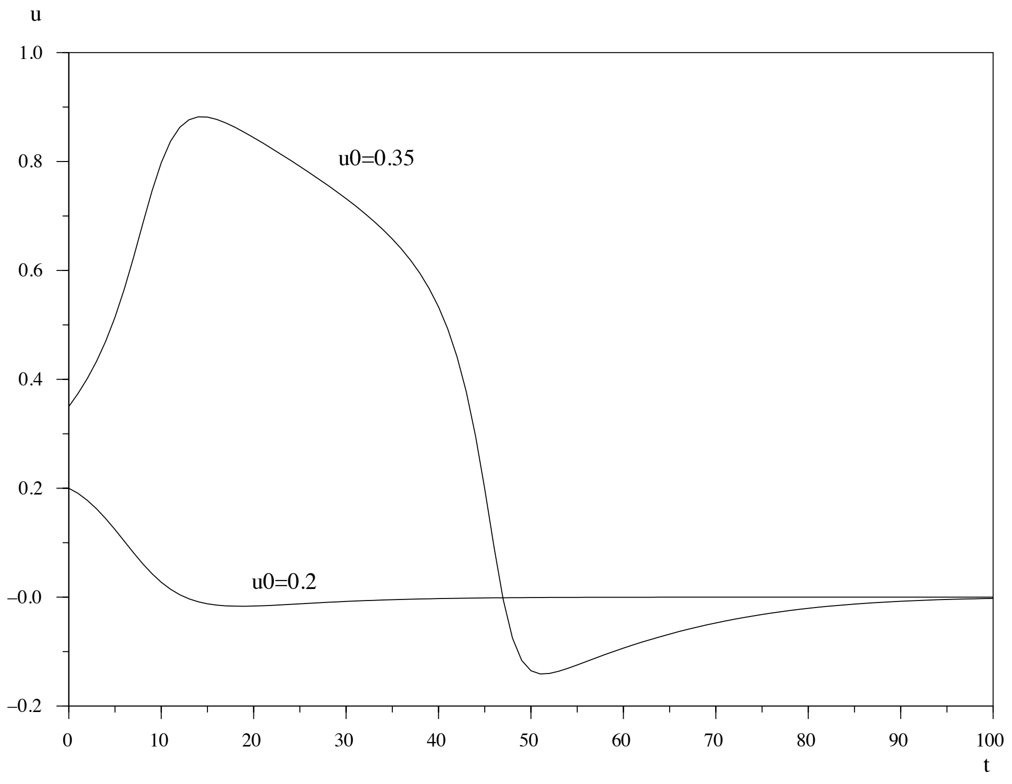

✓ Poor qualitative solution : overshoot, stiff repolarization.

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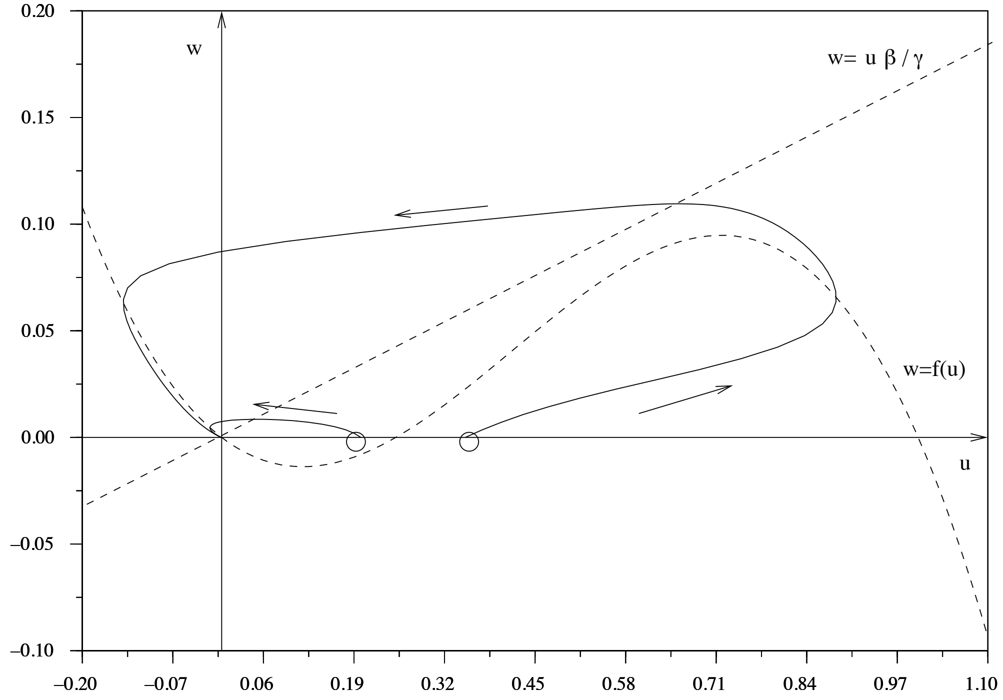

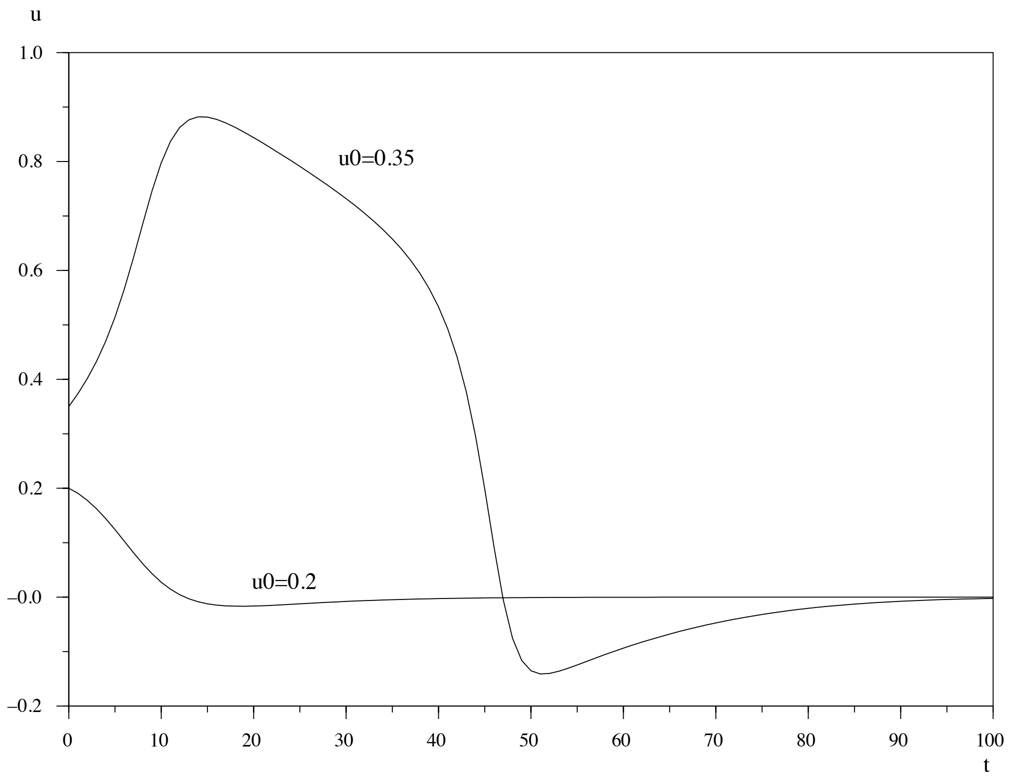

- ✓ Poor qualitative solution : overshoot, stiff repolarization.
- ✓ Difficult to set the four phases of the cardiac cycle *(initiation, plateau, decay, recovery)*

Mitchell-Schaeffer's model

$$\begin{cases} C_\mathrm{m}\dfrac{\mathrm{d}V_\mathrm{m}}{\mathrm{d}t} + I_\mathrm{ion}(V_\mathrm{m}, \boldsymbol{g}) = 0 \\ \dfrac{\mathrm{d}\boldsymbol{g}}{\mathrm{d}t} + G(V_\mathrm{m}, \boldsymbol{g}) = \mathbf{0} \end{cases}$$

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(Mitchell, Schaeffer 2003-04)

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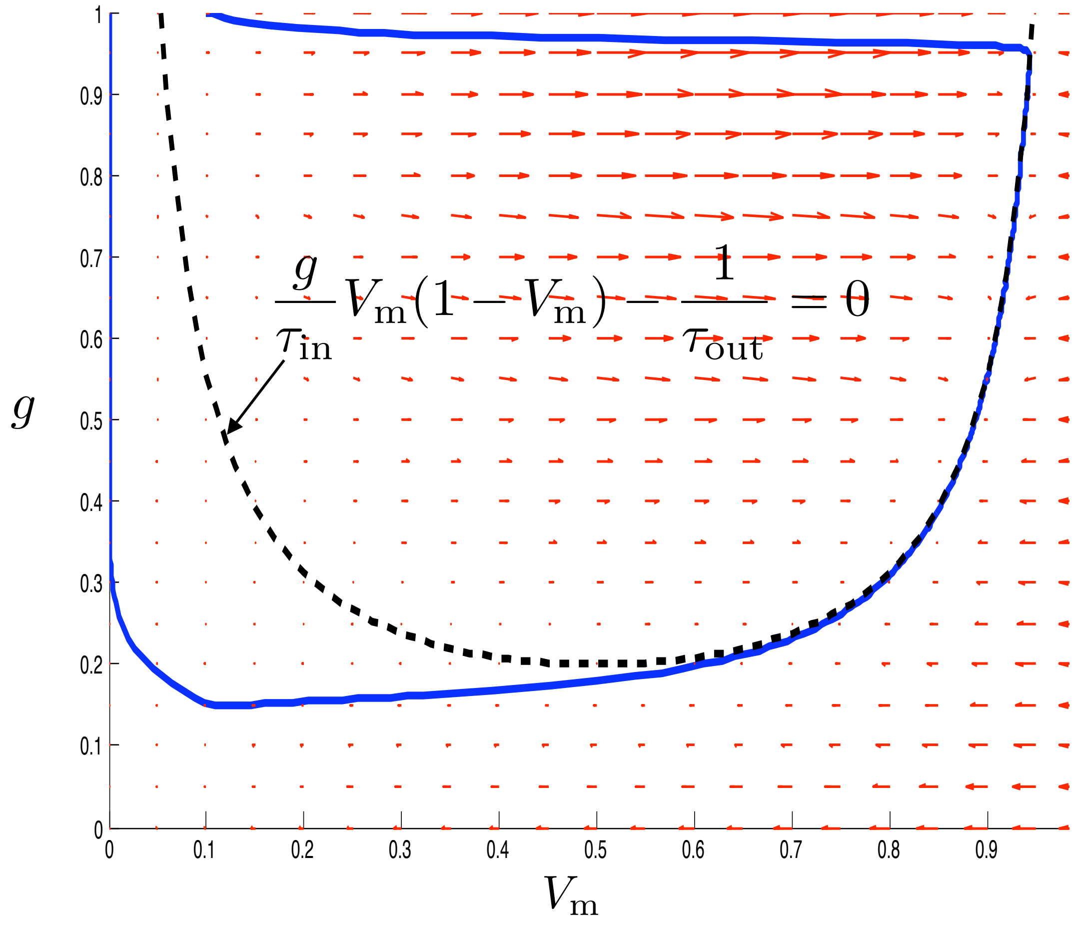
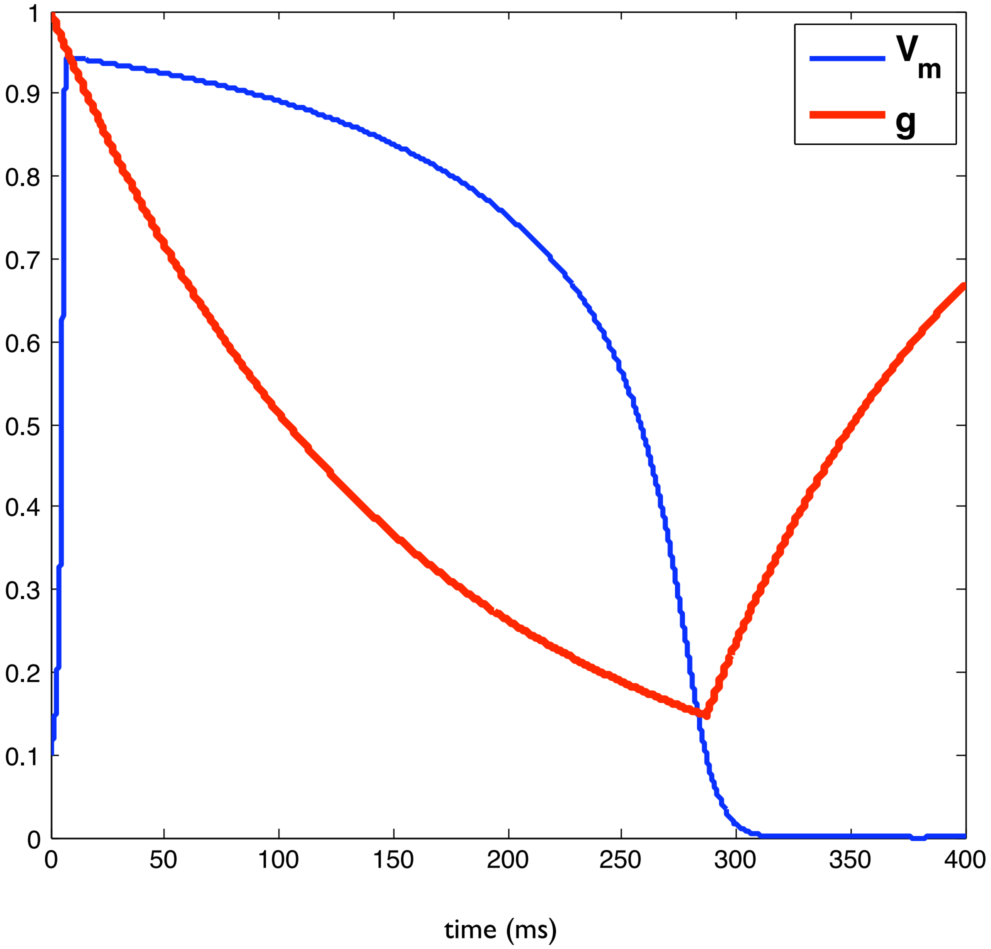
Mitchell-Schaeffer's Action Potential



phase Outlinee(V_m, g)



(V_m, g) vs. time

$$C_\mathrm{m} \frac{\mathrm{d}V_\mathrm{m}}{\mathrm{d}t} = \frac{g}{\tau_\mathrm{in}} V_\mathrm{m}^2 (1 - V_\mathrm{m}) - \frac{V_\mathrm{m}}{\tau_\mathrm{out}}$$

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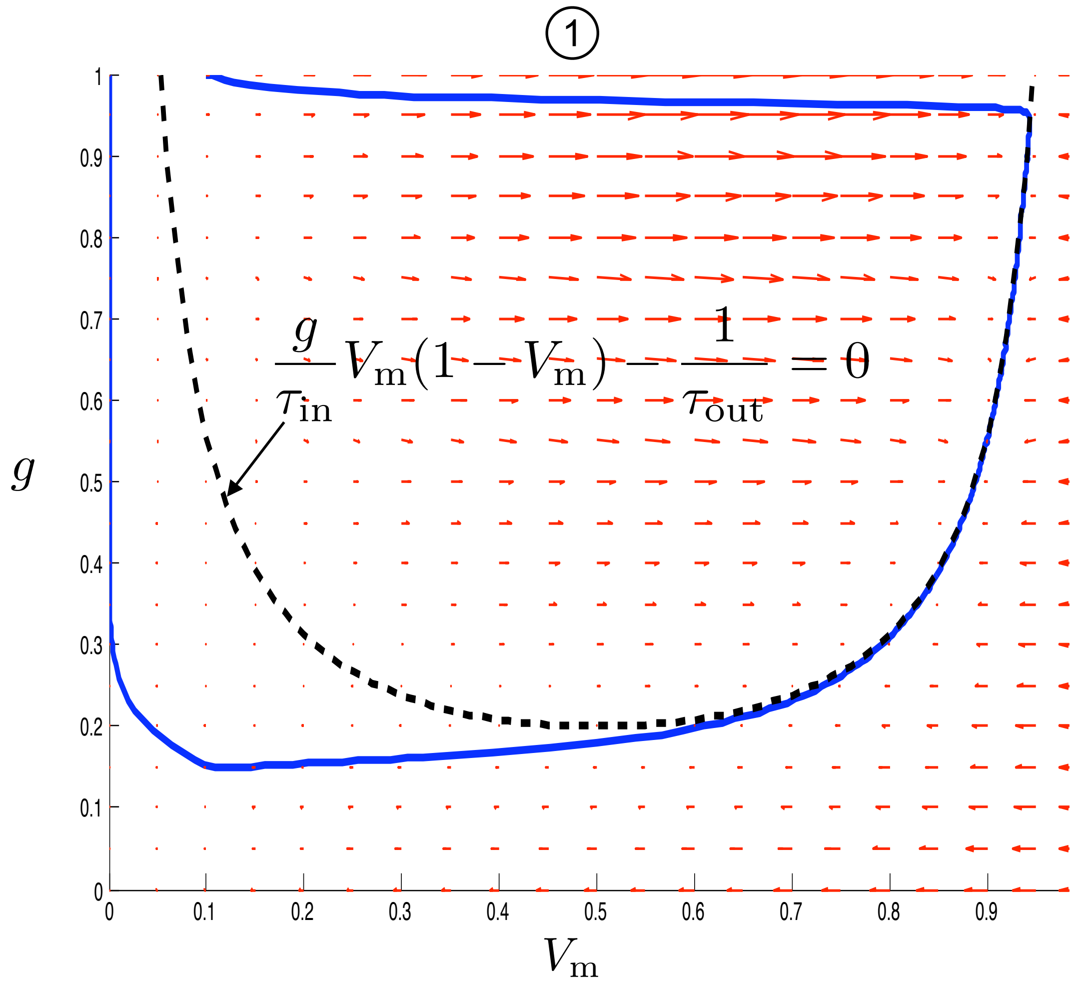

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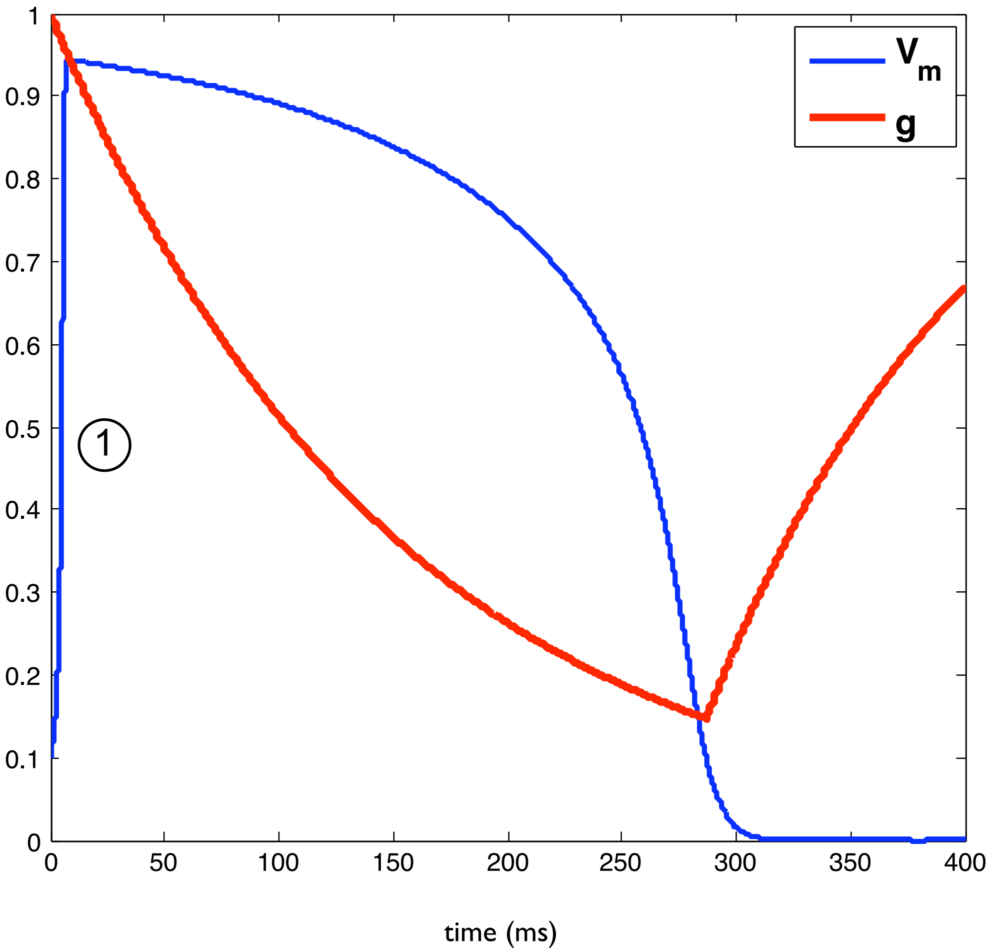

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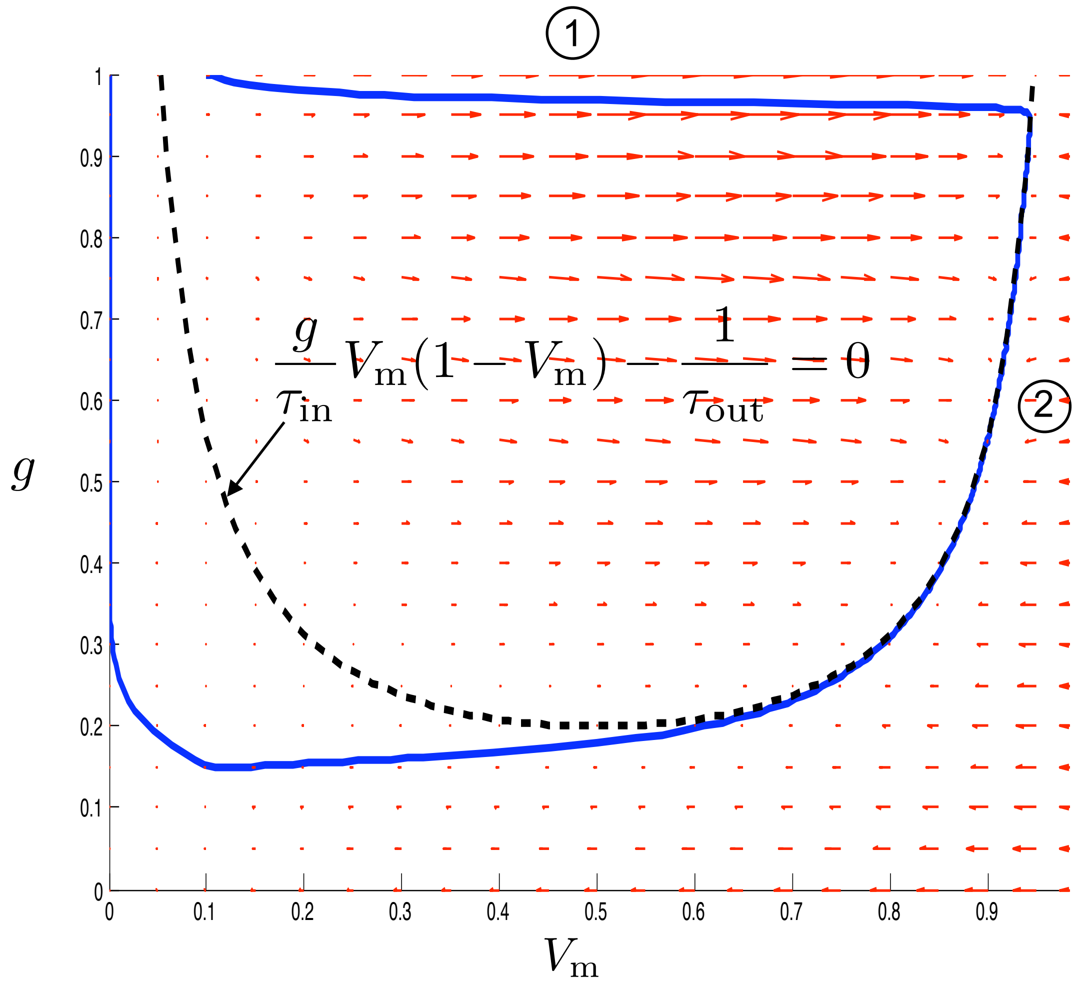
① Depolarization (time scale τ_in)

Mitchell-Schaeffer’s Action Potential



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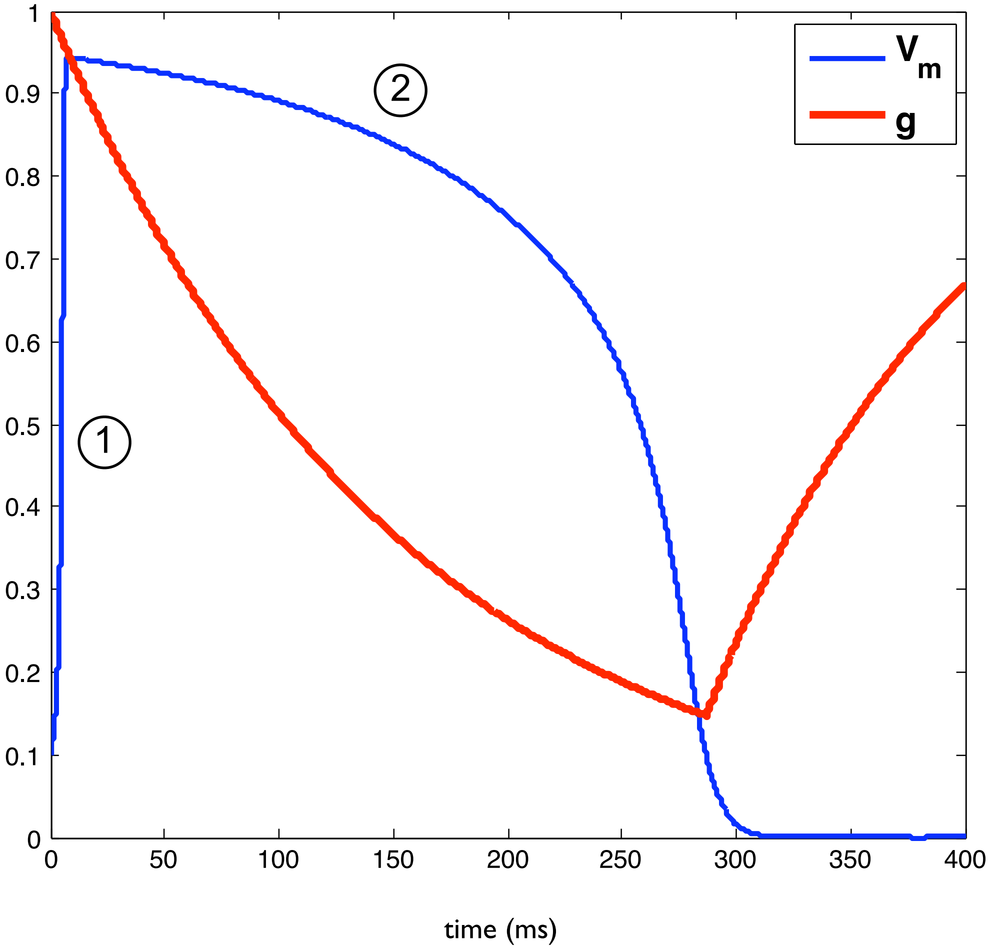

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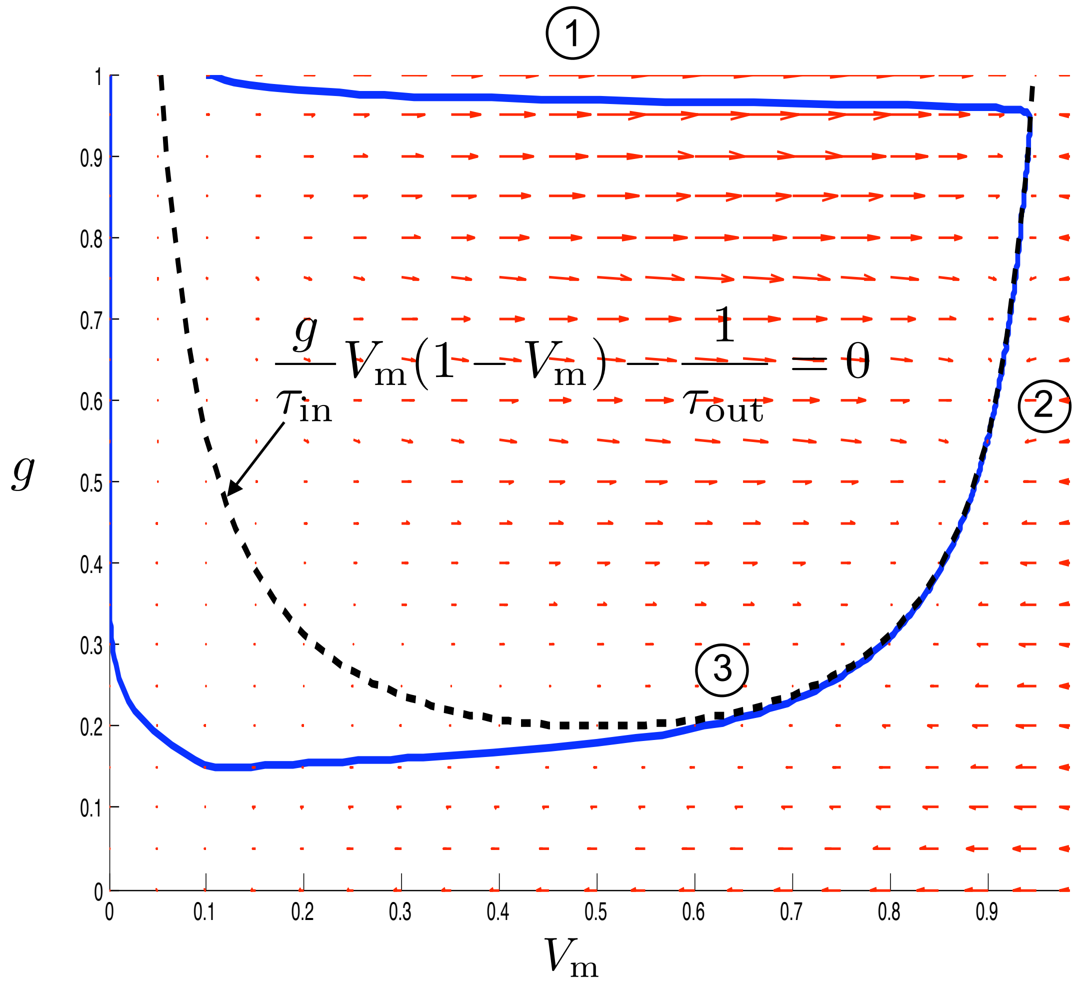
① Depolarization (time scale τ_in)

② Plateau phase (time scale τ_close)

Mitchell-Schaeffer's Action Potential



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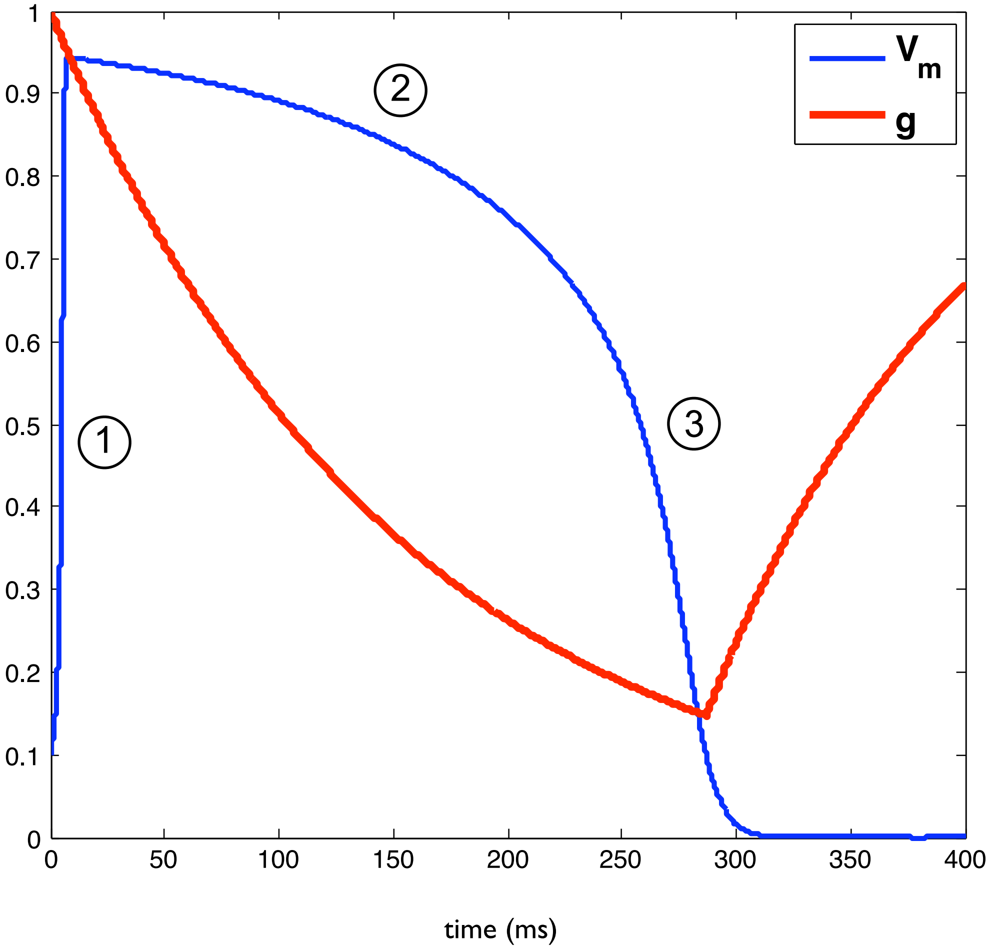

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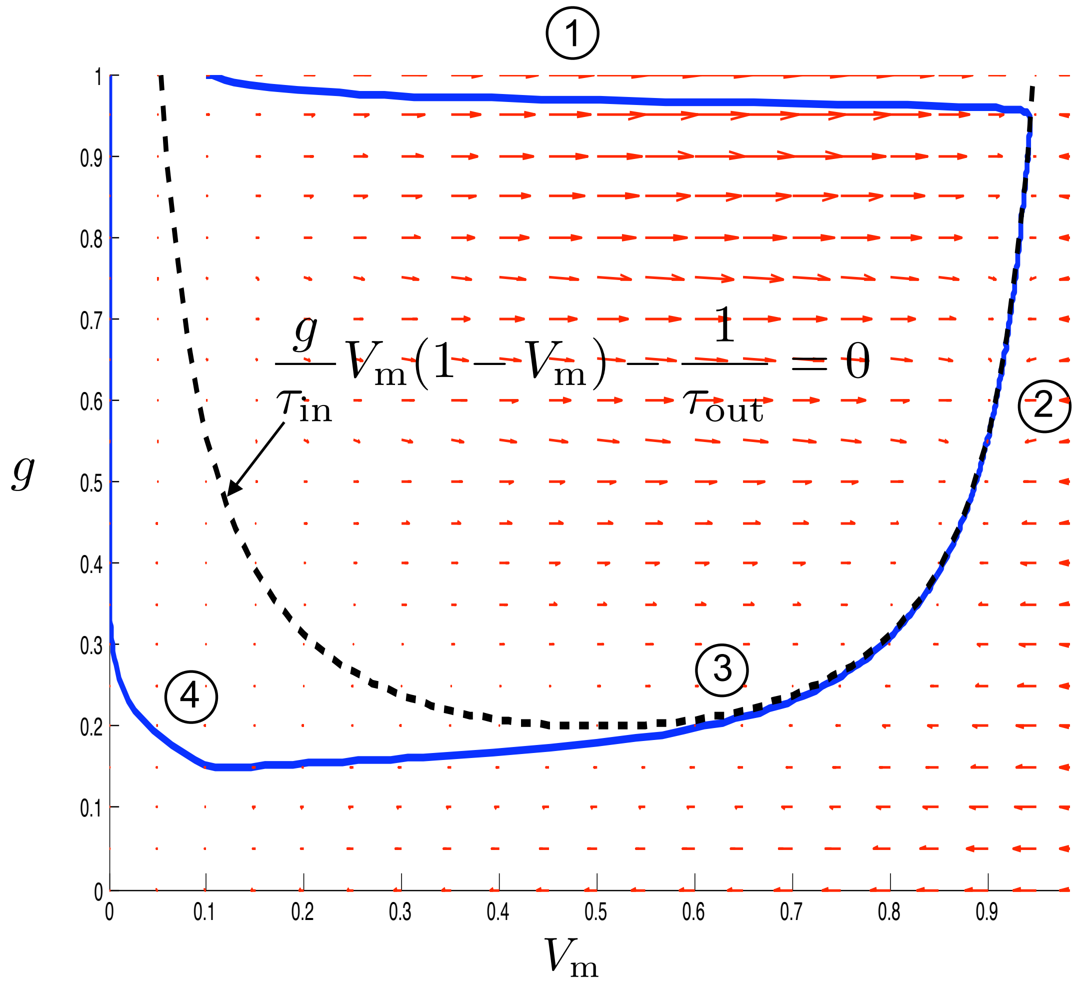
① Depolarization (time scale τ_{in})

② Plateau phase (time scale τ_{close})

③ Repolarization (time scale τ_{out})

Mitchell-Schaeffer's Action Potential



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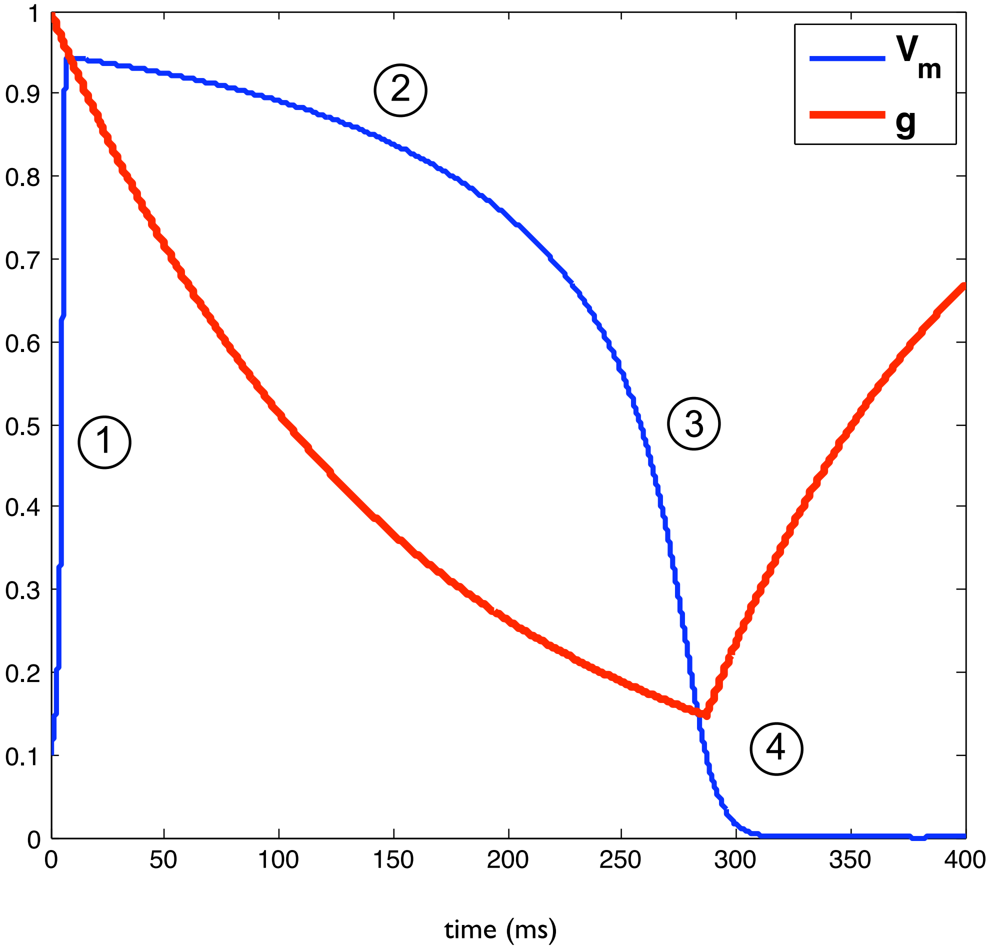

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① Depolarization (time scale τ_in)

② Plateau phase (time scale τ_close)

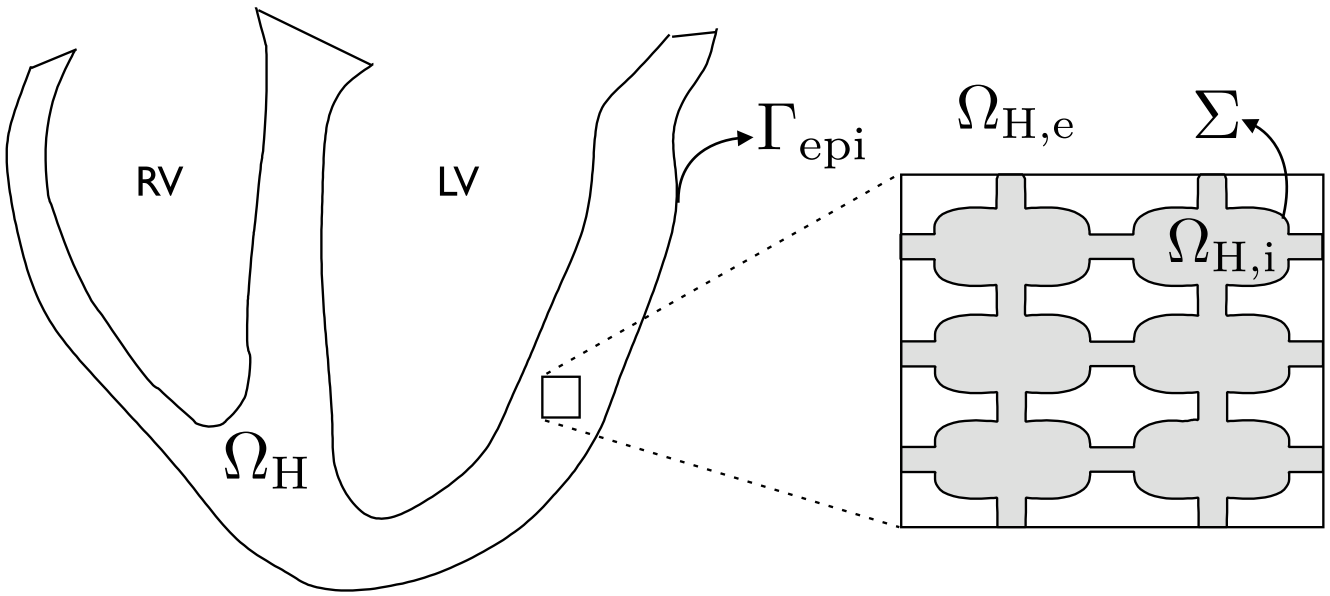
③ Repolarization (time scale τ_out)

④ Recovery (time scale τ_open)

Outline

- Electrical activity of the heart
  - Electrocardiograms (ECG)
  - Cell scale
  - **Tissue scale : the bidomain equations**
  - ECG simulation
- Applications
  - Cardiac Resynchronisation Therapy
  - MRI, Magnetohydrodynamics & ECG

Tissue scale



$\Omega_{\rm H,i}$ intra-cellular medium

$\Omega_{\rm H,e}$ extra-cellular medium

Σ membrane

$\boldsymbol{\sigma}_{\rm i}$, $\boldsymbol{\sigma}_{\rm e}$ intra and extra-cellular conductivity

Electrical balance

$$\begin{aligned}\mathrm{div}(\boldsymbol{\sigma}_{\rm i}\boldsymbol{\nabla}u_{\rm i}) = 0, &\quad \text{in} \quad \Omega_{\rm H,i}\\ \mathrm{div}(\boldsymbol{\sigma}_{\rm e}\boldsymbol{\nabla}u_{\rm e}) = 0, &\quad \text{in} \quad \Omega_{\rm H,e}\end{aligned}$$

$$\left.\begin{aligned}&\boldsymbol{\sigma}_{\rm i}\boldsymbol{\nabla}u_{\rm i}\cdot\boldsymbol{n} = \boldsymbol{\sigma}_{\rm e}\boldsymbol{\nabla}u_{\rm e}\cdot\boldsymbol{n}\\ &\boldsymbol{\sigma}_{\rm i}\boldsymbol{\nabla}u_{\rm i}\cdot\boldsymbol{n} = C_{\rm m}\frac{\partial V_{\rm m}}{\partial t} + I_{\rm ion}(V_{\rm m},\boldsymbol{g})\end{aligned}\right\}\quad \text{on} \quad \Sigma$$

Tissue scale : homogenization

Microscopic scale

Tissue scale

(Tung 78, Krassowska-Neu 93, Colli Franzone-Savaré 02)

Tissue scale : homogenization

Microscopic scale

$\Omega_{\mathrm{H,i}}$ and $\Omega_{\mathrm{H,e}}$

Tissue scale

$\Omega_{\mathrm{H}} = \Omega_{\mathrm{H,i}} = \Omega_{\mathrm{H,e}}$

(Tung 78, Krassowska-Neu 93, Colli Franzone-Savaré 02)

Tissue scale : homogenization

Microscopic scale

$\Omega_{\mathrm{H,i}}$ and $\Omega_{\mathrm{H,e}}$

$$\boldsymbol{\sigma}_{\mathrm{i}} \boldsymbol{\nabla} \phi_{\mathrm{i}} \cdot \boldsymbol{n} = \boldsymbol{\sigma}_{\mathrm{e}} \boldsymbol{\nabla} \phi_{\mathrm{e}} \cdot \boldsymbol{n}$$

Tissue scale

$$\Omega_{\mathrm{H}} = \Omega_{\mathrm{H,i}} = \Omega_{\mathrm{H,e}}$$

$$-\operatorname{div}(\boldsymbol{\sigma}_{\mathrm{i}} \boldsymbol{\nabla} \phi_{\mathrm{i}}) = \operatorname{div}(\boldsymbol{\sigma}_{\mathrm{e}} \boldsymbol{\nabla} \phi_{\mathrm{e}})$$

(Tung 78, Krassowska-Neu 93, Colli Franzone-Savaré 02)

Tissue scale : homogenization

Microscopic scale

$$\Omega_{\mathrm{H,i}} \text{ and } \Omega_{\mathrm{H,e}}$$

$$\boldsymbol{\sigma}_{\mathrm{i}} \boldsymbol{\nabla} \phi_{\mathrm{i}} \cdot \boldsymbol{n} = \boldsymbol{\sigma}_{\mathrm{e}} \boldsymbol{\nabla} \phi_{\mathrm{e}} \cdot \boldsymbol{n}$$

$$C_{\mathrm{m}} \frac{\partial V_{\mathrm{m}}}{\partial t} + I_{\mathrm{ion}} = -\boldsymbol{\sigma}_{\mathrm{i}} \boldsymbol{\nabla} \phi_{\mathrm{i}} \cdot \boldsymbol{n}$$

Tissue scale

$$\Omega_{\mathrm{H}} = \Omega_{\mathrm{H,i}} = \Omega_{\mathrm{H,e}}$$

$$-\operatorname{div}(\boldsymbol{\sigma}_{\mathrm{i}} \boldsymbol{\nabla} \phi_{\mathrm{i}}) = \operatorname{div}(\boldsymbol{\sigma}_{\mathrm{e}} \boldsymbol{\nabla} \phi_{\mathrm{e}})$$

$$A_{\mathrm{m}} \left(C_{\mathrm{m}} \frac{\partial V_{\mathrm{m}}}{\partial t} + I_{\mathrm{ion}} \right) = \operatorname{div}(\boldsymbol{\sigma}_{\mathrm{i}} \boldsymbol{\nabla} \phi_{\mathrm{i}})$$

(Tung 78, Krassowska-Neu 93, Colli Franzone-Savaré 02)

Tissue scale

- Bidomain equations :

$$\begin{cases} A_{\mathrm{m}} \left(C_{\mathrm{m}} \dfrac{\partial V_{\mathrm{m}}}{\partial t} + I_{\mathrm{ion}}(V_{\mathrm{m}}, \boldsymbol{g}) \right) - \mathrm{div}(\boldsymbol{\sigma}_{\mathrm{i}} \boldsymbol{\nabla} u_{\mathrm{i}}) = A_{\mathrm{m}} I_{\mathrm{app}}, & \text{in} \quad \Omega_{\mathrm{H}} \\ \mathrm{div}(\boldsymbol{\sigma}_{\mathrm{e}} \boldsymbol{\nabla} u_{\mathrm{e}}) = - \mathrm{div}(\boldsymbol{\sigma}_{\mathrm{i}} \boldsymbol{\nabla} u_{\mathrm{i}}), & \text{in} \quad \Omega_{\mathrm{H}} \\ \dfrac{\partial \boldsymbol{g}}{\partial t} + G(V_{\mathrm{m}}, \boldsymbol{g}) = 0, & \text{in} \quad \Omega_{\mathrm{H}} \\ \boldsymbol{\sigma}_{\mathrm{i}} \boldsymbol{\nabla} u_{\mathrm{i}} \cdot \boldsymbol{n} = 0, & \text{on} \quad \Gamma_{\mathrm{epi}} \\ \boldsymbol{\sigma}_{\mathrm{e}} \boldsymbol{\nabla} u_{\mathrm{e}} \cdot \boldsymbol{n} = 0, & \text{on} \quad \Gamma_{\mathrm{epi}} \end{cases}$$

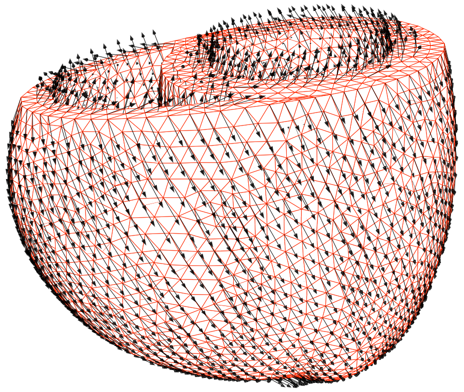
Tissue scale

- Bidomain equations :

$$\left\{\begin{aligned} A_{\mathrm{m}}\left(C_{\mathrm{m}}\frac{\partial V_{\mathrm{m}}}{\partial t}+I_{\mathrm{ion}}(V_{\mathrm{m}},\boldsymbol{g})\right)-\operatorname{div}(\boldsymbol{\sigma}_{\mathrm{i}}\boldsymbol{\nabla}u_{\mathrm{i}})=A_{\mathrm{m}}I_{\mathrm{app}}, &\quad \text{in} \quad \Omega_{\mathrm{H}}\\ \operatorname{div}(\boldsymbol{\sigma}_{\mathrm{e}}\boldsymbol{\nabla}u_{\mathrm{e}})=-\operatorname{div}(\boldsymbol{\sigma}_{\mathrm{i}}\boldsymbol{\nabla}u_{\mathrm{i}}), &\quad \text{in} \quad \Omega_{\mathrm{H}}\\ \frac{\partial \boldsymbol{g}}{\partial t}+G(V_{\mathrm{m}},\boldsymbol{g})=0, &\quad \text{in} \quad \Omega_{\mathrm{H}}\\ \boldsymbol{\sigma}_{\mathrm{i}}\boldsymbol{\nabla}u_{\mathrm{i}}\cdot\boldsymbol{n}=0, &\quad \text{on} \quad \Gamma_{\mathrm{epi}}\\ \boldsymbol{\sigma}_{\mathrm{e}}\boldsymbol{\nabla}u_{\mathrm{e}}\cdot\boldsymbol{n}=0, &\quad \text{on} \quad \Gamma_{\mathrm{epi}} \end{aligned}\right.$$

- Anisotropic conductivity

$$\boldsymbol{\sigma}_{\mathrm{i},e}(x)=\sigma^{\mathrm{t}}_{\mathrm{i},e}I+(\sigma^{\mathrm{l}}_{\mathrm{i},e}-\sigma^{\mathrm{t}}_{\mathrm{i},e})\boldsymbol{a}(x)\otimes\boldsymbol{a}(x)$$



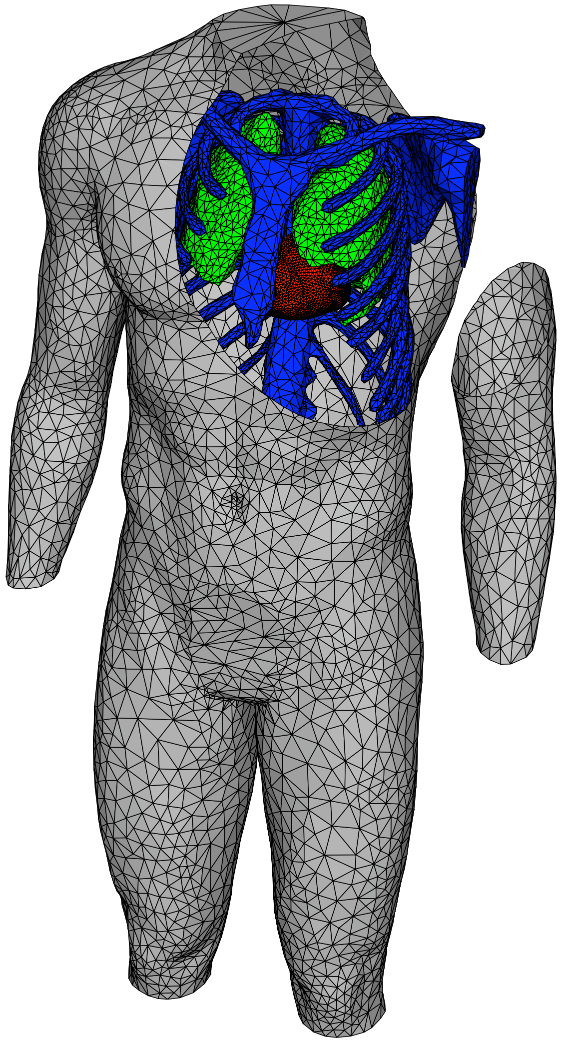
- If the anisotropy is the same in both media : mono-domain equations

Heart-torso coupling

- Bidomain model in the heart

- Torso:

$$\begin{cases} \operatorname{div}(\boldsymbol{\sigma}_{\mathrm{T}} \boldsymbol{\nabla} u_{\mathrm{T}}) = 0, & \text{in} \quad \Omega_{\mathrm{T}} \\ \boldsymbol{\sigma}_{\mathrm{T}} \boldsymbol{\nabla} u_{\mathrm{T}} \cdot \boldsymbol{n}_{\mathrm{T}} = 0, & \text{on} \quad \Gamma_{\mathrm{ext}} \end{cases}$$

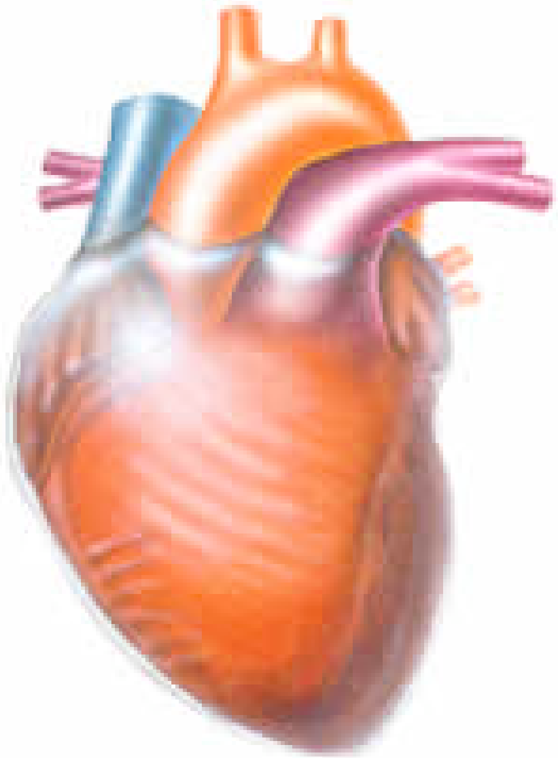


- Coupling conditions:

$$\begin{cases} u_{\mathrm{e}} = u_{\mathrm{T}}, & \text{on} \quad \Gamma_{\mathrm{epi}} \\ \boldsymbol{\sigma}_{\mathrm{e}} \boldsymbol{\nabla} u_{\mathrm{e}} \cdot \boldsymbol{n} = \boldsymbol{\sigma}_{\mathrm{T}} \boldsymbol{\nabla} u_{T} \cdot \boldsymbol{n}, & \text{on} \quad \Gamma_{\mathrm{epi}} \end{cases}$$

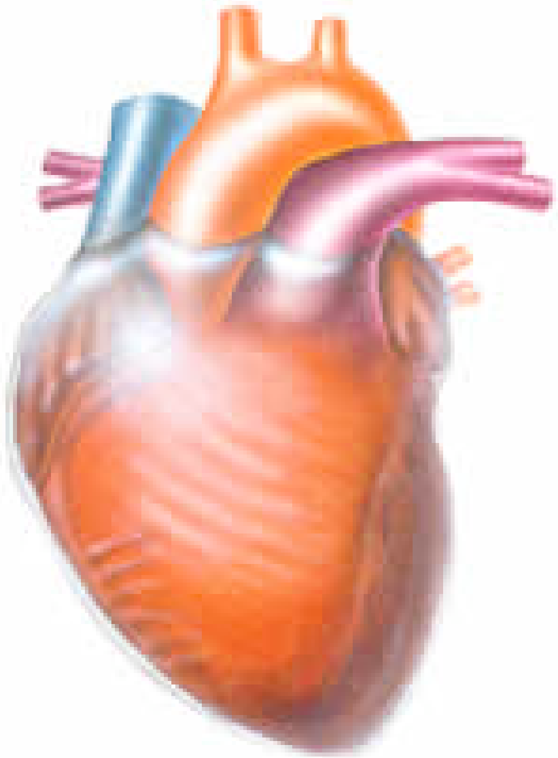
(Krassowsca-Neu 94, Clements et al. 04, Pierre 05, Lines et. al 06,...)

Other heart-torso coupling conditions



- A possible simplification:

Other heart-torso coupling conditions

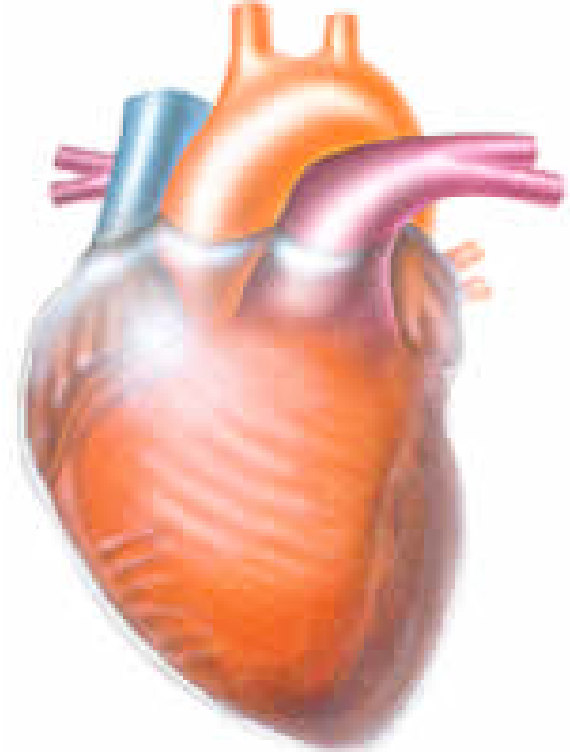


- A possible simplification:

  If we neglect the impact of the torso:

$$\begin{cases} u_{\mathrm{e}} = u_{\mathrm{T}}, & \text{on} \quad \Gamma_{\mathrm{epi}} \\ \boldsymbol{\sigma}_{\mathrm{e}} \boldsymbol{\nabla} u_{\mathrm{e}} \cdot \boldsymbol{n} = 0, & \text{on} \quad \Gamma_{\mathrm{epi}} \end{cases}$$

Other heart-torso coupling conditions



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- Pericardium resistive-capacitor effect:

$$\begin{cases} \boldsymbol{\sigma}_{\mathrm{e}} \nabla u_{\mathrm{e}} \cdot \boldsymbol{n} = C_{\mathrm{peri}} \frac{\partial}{\partial t}(u_{\mathrm{T}} - u_{\mathrm{e}}) + \frac{1}{R_{\mathrm{peri}}}(u_{\mathrm{T}} - u_{\mathrm{e}}), & \text{on} \quad \Gamma_{\mathrm{epi}} \\ \boldsymbol{\sigma}_{\mathrm{T}} \nabla u_{\mathrm{T}} \cdot \boldsymbol{n} = \boldsymbol{\sigma}_{\mathrm{e}} \nabla u_{\mathrm{e}} \cdot \boldsymbol{n}, & \text{on} \quad \Gamma_{\mathrm{epi}} \end{cases}$$

R_{peri} : pericardium resistance

C_{peri} : pericardium capacitance

(Boulakia-Fernandez-JFG-Zemzemi FIMH07)

The heart-torso coupled problem

- $V_{\mathrm{m}} = u_{\mathrm{i}} - u_{\mathrm{e}}$

$$\left\{\begin{aligned} A_{\mathrm{m}}\left(C_{\mathrm{m}}\frac{\partial V_{\mathrm{m}}}{\partial t} + I_{\mathrm{ion}}(V_{\mathrm{m}}, \boldsymbol{g})\right) - \mathrm{div}(\boldsymbol{\sigma}_{\mathrm{i}}\boldsymbol{\nabla} u_{\mathrm{i}}) &= I_{\mathrm{app}}, \quad \text{in} \quad \Omega_{\mathrm{H}}\\ \mathrm{div}(\boldsymbol{\sigma}_{\mathrm{e}}\boldsymbol{\nabla} u_{\mathrm{e}}) &= -\,\mathrm{div}(\boldsymbol{\sigma}_{\mathrm{i}}\boldsymbol{\nabla} u_{\mathrm{i}}), \quad \text{in} \quad \Omega_{\mathrm{H}}\\ \boldsymbol{\sigma}_{\mathrm{i}}\boldsymbol{\nabla} u_{\mathrm{i}} \cdot \boldsymbol{n} &= 0, \quad \text{on} \quad \Gamma_{\mathrm{epi}}\\ \frac{\partial \boldsymbol{g}}{\partial t} + G(V_{\mathrm{m}}, \boldsymbol{g}) &= 0, \quad \text{in} \quad \Omega_{\mathrm{H}}\\ \mathrm{div}(\boldsymbol{\sigma}_{\mathrm{T}}\boldsymbol{\nabla} u_{\mathrm{T}}) &= 0, \quad \text{in} \quad \Omega_{\mathrm{T}} \end{aligned}\right.$$

$$\left\{\begin{aligned} u_{\mathrm{e}} &= u_{\mathrm{T}}, && \text{on} \quad \Gamma_{\mathrm{epi}}\\ \boldsymbol{\sigma}_{\mathrm{e}}\boldsymbol{\nabla} u_{\mathrm{e}} \cdot \boldsymbol{n} &= \boldsymbol{\sigma}_{\mathrm{T}}\boldsymbol{\nabla} u_{\mathrm{T}} \cdot \boldsymbol{n}, && \text{on} \quad \Gamma_{\mathrm{epi}} \end{aligned}\right.$$

The heart-torso coupled problem

- $V_\mathrm{m} = u_\mathrm{i} - u_\mathrm{e}$

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$$\begin{cases} u_\mathrm{e} = u_\mathrm{T}, & \text{on} \quad \Gamma_\mathrm{epi} \\ \boldsymbol{\sigma}_\mathrm{e}\boldsymbol{\nabla} u_\mathrm{e}\cdot\boldsymbol{n} = \boldsymbol{\sigma}_\mathrm{T}\boldsymbol{\nabla} u_\mathrm{T}\cdot\boldsymbol{n}, & \text{on} \quad \Gamma_\mathrm{epi} \end{cases}$$

- It involves:
  - 2 degenerated non-linear reaction-diffusion equations
  - a system of ODE’s and a diffusion equation

The heart-torso coupled problem

- $V_{\mathrm{m}} = u_{\mathrm{i}} - u_{\mathrm{e}}$

$$\begin{cases} A_{\mathrm{m}} \left(C_{\mathrm{m}} \dfrac{\partial V_{\mathrm{m}}}{\partial t} + I_{\mathrm{ion}}(V_{\mathrm{m}}, \boldsymbol{g}) \right) - \mathrm{div}(\boldsymbol{\sigma}_{\mathrm{i}} \boldsymbol{\nabla} V_{\mathrm{m}}) - \mathrm{div}(\boldsymbol{\sigma}_{\mathrm{i}} \boldsymbol{\nabla} u_{\mathrm{e}}) = I_{\mathrm{app}}, \quad \text{in} \quad \Omega_{\mathrm{H}} \\ \mathrm{div}(\boldsymbol{\sigma}_{\mathrm{i}} \boldsymbol{\nabla} V_{\mathrm{m}}) + \mathrm{div}\big((\boldsymbol{\sigma}_{\mathrm{i}} + \boldsymbol{\sigma}_{\mathrm{e}}) \boldsymbol{\nabla} u_{\mathrm{e}} \big) = 0, \quad \text{in} \quad \Omega_{\mathrm{H}} \\ \boldsymbol{\sigma}_{\mathrm{i}} \boldsymbol{\nabla} V_{\mathrm{m}} \cdot \boldsymbol{n} + \boldsymbol{\sigma}_{\mathrm{i}} \boldsymbol{\nabla} u_{\mathrm{e}} \cdot \boldsymbol{n} = 0, \quad \text{on} \quad \Gamma_{\mathrm{epi}} \\ \dfrac{\partial \boldsymbol{g}}{\partial t} + G(V_{\mathrm{m}}, \boldsymbol{g}) = 0, \quad \text{in} \quad \Omega_{\mathrm{H}} \\ \mathrm{div}(\boldsymbol{\sigma}_{\mathrm{T}} \boldsymbol{\nabla} u_{\mathrm{T}}) = 0, \quad \text{in} \quad \Omega_{\mathrm{T}} \end{cases}$$

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Well-posedness analysis

- Isolated bidomain model (no coupling with torso):
  Colli Franzone et al. 02, Bendahmane et al. 06, Bourgault et al. 06, Veneroni 06

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    - Generalized FitzHugh-Nagumo:
      $$I_{\text{ion}}(V_{\text{m}}, g) = f_1(V_{\text{m}}) + f_2(V_{\text{m}})g$$
      $$G(V_{\text{m}}, g) = g_1(V_{\text{m}}) + g_2 g$$
    - Regularized Mitchell-Schaeffer

$$I_{\text{ion}}(V_{\text{m}}, g) = -\frac{g}{\tau_{\text{in}}} f_1(v) + \frac{V_{\text{m}}}{\tau_{\text{out}}}, \quad G(V_{\text{m}}, g) = \left(\frac{1}{\tau_{\text{close}}} + \frac{\tau_{\text{close}} - \tau_{\text{open}}}{\tau_{\text{close}}\tau_{\text{open}}} h_\infty(V_{\text{m}}) \right) \left(g - h_\infty(V_{\text{m}}) \right)$$

$$h_\infty(v) = \frac{1}{2} \left[1 - \tanh\left(\frac{v - V_{\text{cr}}}{\eta} \right) \right], \quad \eta > 0$$

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Theorem: *(Boulakia-Fernandez-JFG-Zemzemi 07)*

Under the assumptions
$$\begin{cases} |f_1(v)| \leq c_1 + c_2|v|^3, & f_1(v)v \geq a|v|^4 - b|v|^2 \\ f_2(v) = p + qv, & |g_1(v)| \leq c_3 + c_4|v|^2 \end{cases}$$
the coupled heart-torso problem has a global weak solution

$$V_{\mathrm{m}} \in L^2(0, T; H^1(\Omega_{\mathrm{H}})), \quad \partial_t V_{\mathrm{m}} \in L^2(0, T; (H^1(\Omega_{\mathrm{H}}))'), \quad g \in H^1(0, T; L^2(\Omega_{\mathrm{H}})), \quad (u_{\mathrm{e}}, u_{\mathrm{T}}) \in L^2(0, T; H^1(\Omega))$$

Uniqueness holds for the FitzHugh-Nagumo model.

Well-posedness analysis

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Theorem: *(Boulakia-Fernandez-JFG-Zemzemi 07)*

$, T; H^1(\Omega))$

Idea of the proof:

1. Faedo-Galerkin to an approximate non-degnerate system: we add
$$\frac{1}{n}\partial_t u_{\rm i}^n, \quad -\frac{1}{n}\partial_t u_{\rm e}^n,$$
2. *A priori* energy estimates independent of *n*
3. Compactness

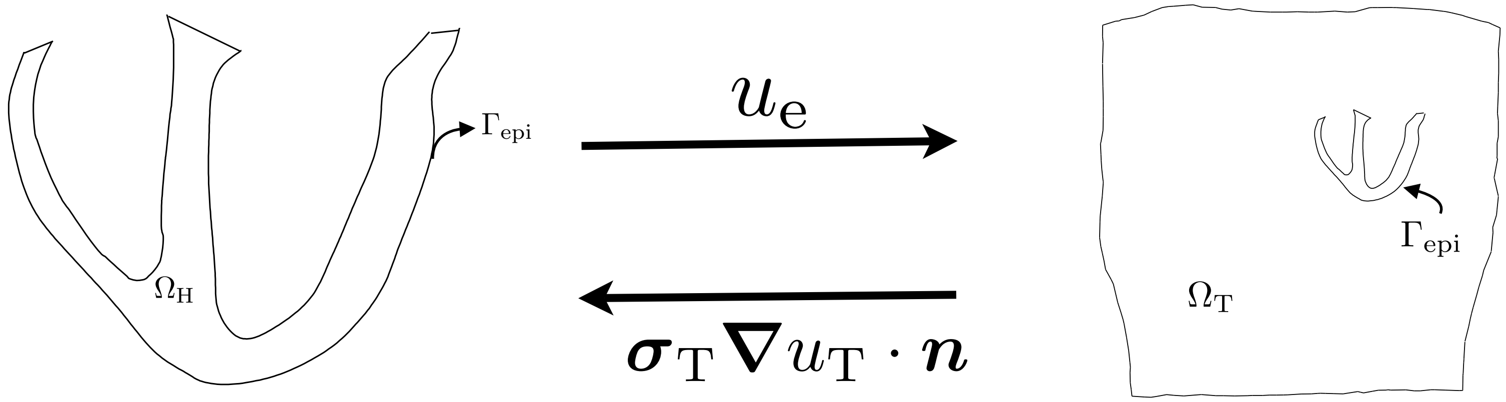
Outline

- Electrical activity of the heart
  - Electrocardiograms (ECG)
  - Cell scale
  - Tissue scale : the bidomain equations
  - **ECG simulation**
- Applications
  - Cardiac Resynchronisation Therapy
  - MRI, Magnetohydrodynamics & ECG

Discretization and solution procedure

- $\mathbb{P}_1$ finite elements in space
- Semi-implicit BDF2 scheme in time
- Coupling with torso:

$$\begin{cases} u_{\mathrm{e}} = u_{\mathrm{T}}, & \text{on} \quad \Gamma_{\mathrm{epi}} \\ \boldsymbol{\sigma}_{\mathrm{e}} \boldsymbol{\nabla} u_{\mathrm{e}} \cdot \boldsymbol{n} = \boldsymbol{\sigma}_{\mathrm{T}} \boldsymbol{\nabla} u_{\mathrm{T}} \cdot \boldsymbol{n}, & \text{on} \quad \Gamma_{\mathrm{epi}} \end{cases}$$



Dirichlet-to-Neumann iterations

“Standard” numerical methods :

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The real difficulty is modelling !

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Options :

- Monodomain or bidomain ?
- Physiologic or phenomenologic ?
- Fibers ?
- Strong coupling with the torso ?
- Cellular heterogeneity ?
- His bundle modelling ? Initialization ?

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The real difficulty is modelling !

Options :

- Monodomain or bidomain ?
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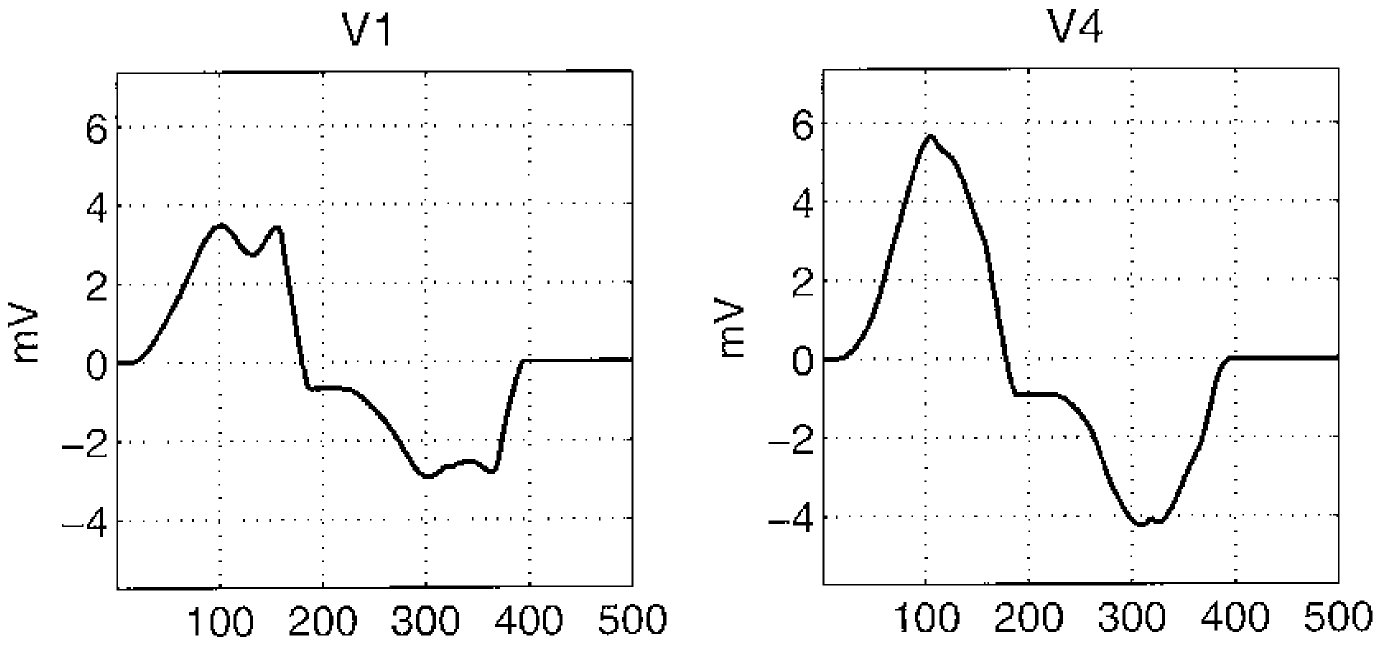
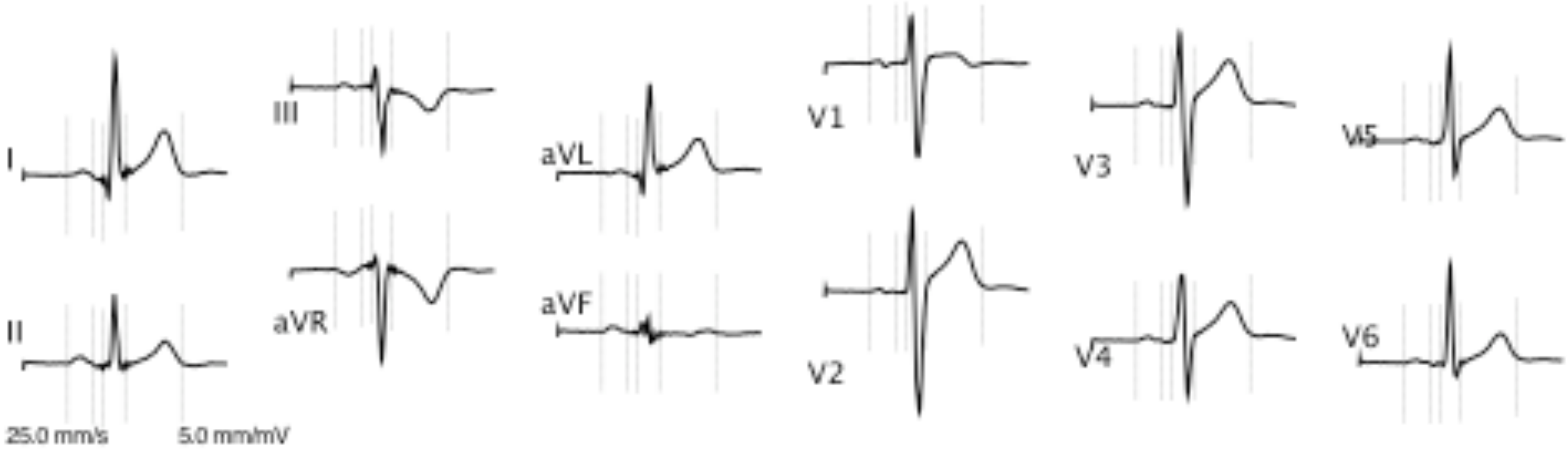
Guideline :

- Devise “a simplest” model which provides a good ECG
- Forget anything which does not affect the ECG

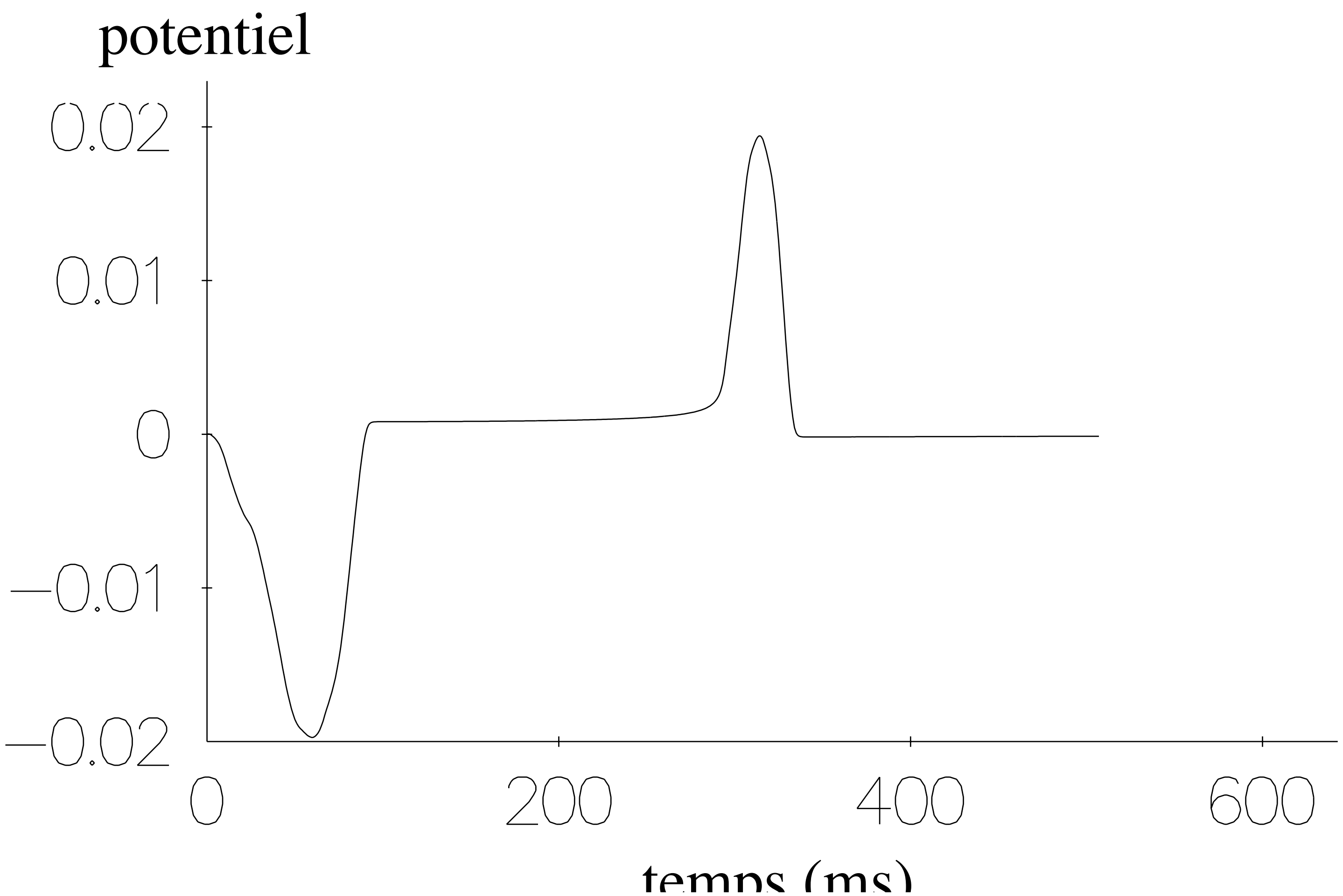
An example from the literature

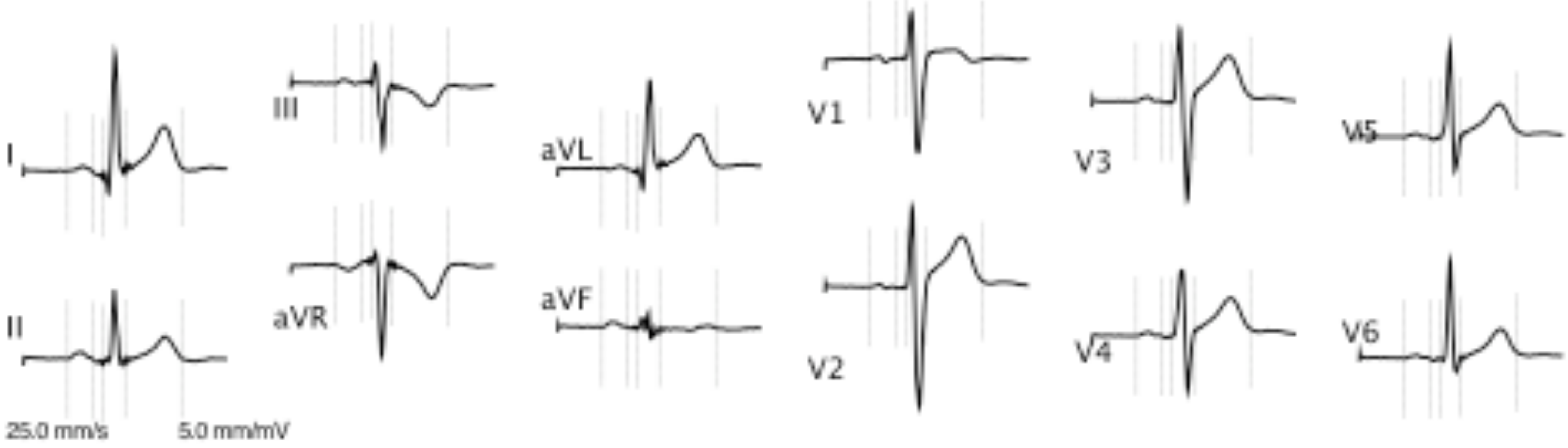


(*Sundnes et al., Springer 2006*)

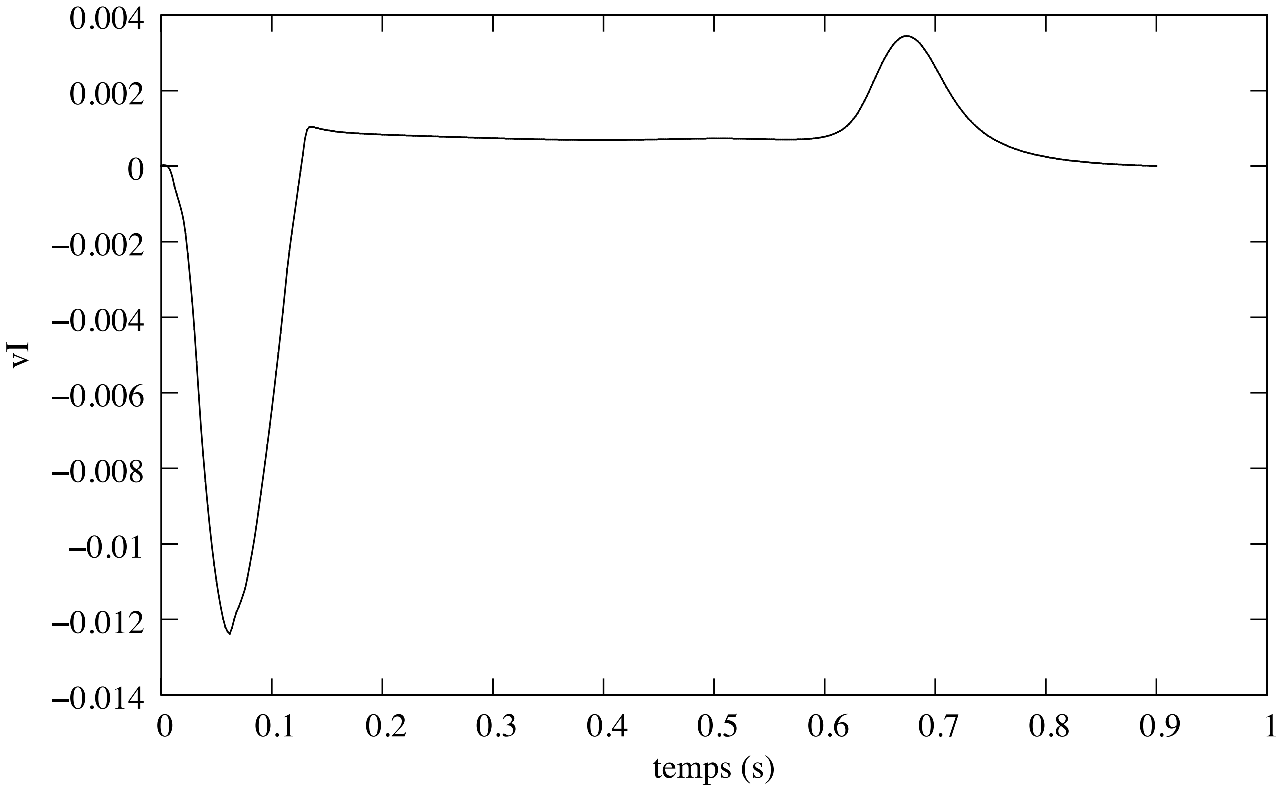


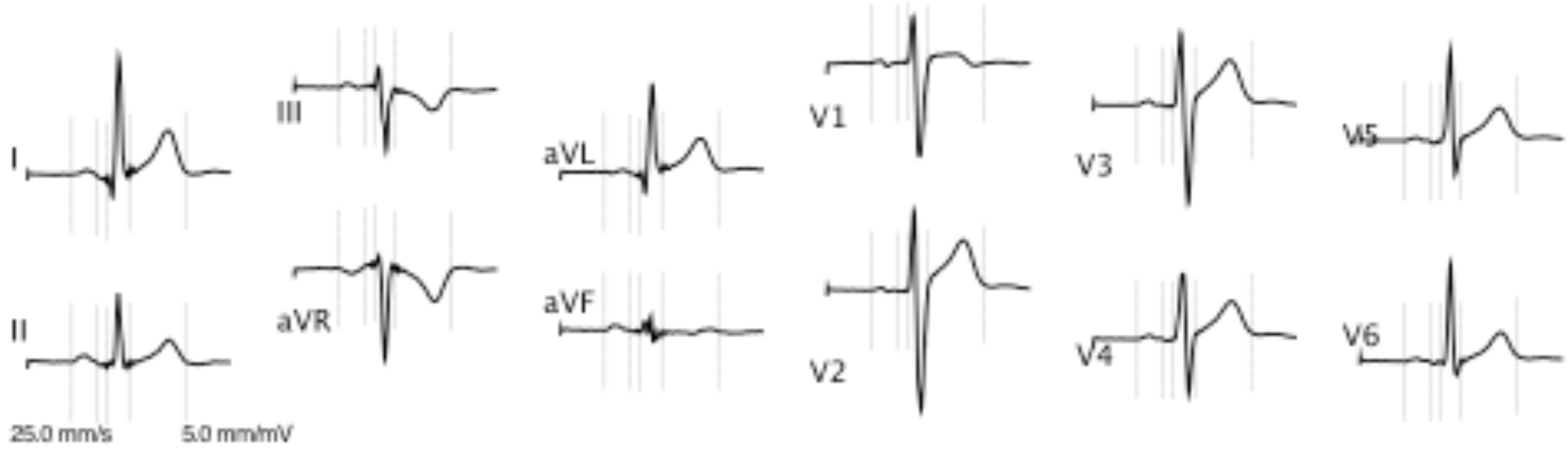
1st example: Fitzhugh-Nagumo





2d example: physiological ionic model



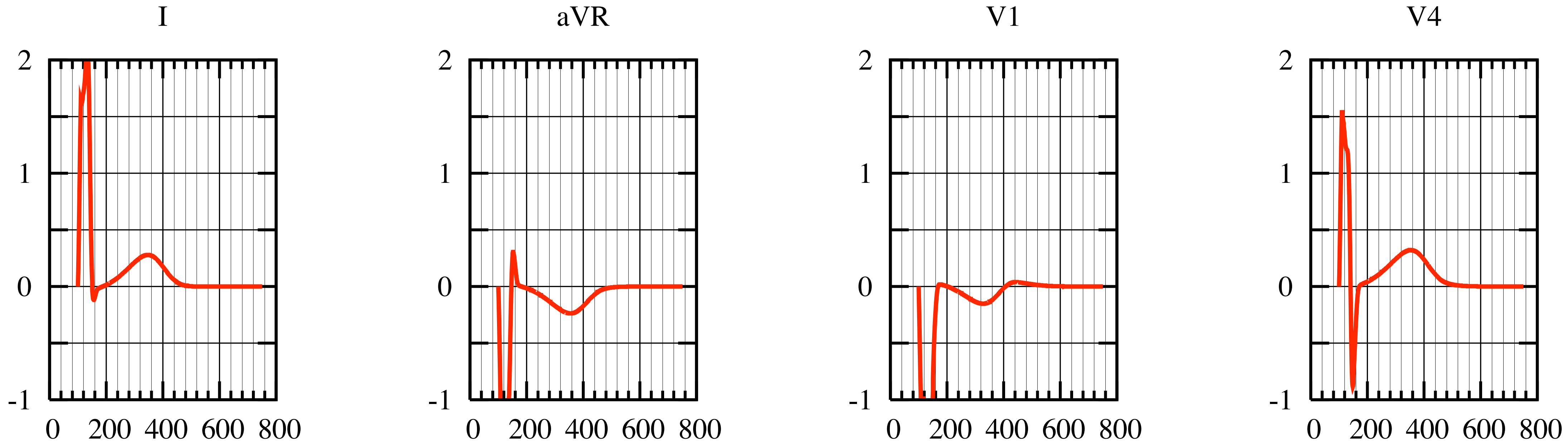


Discussion :

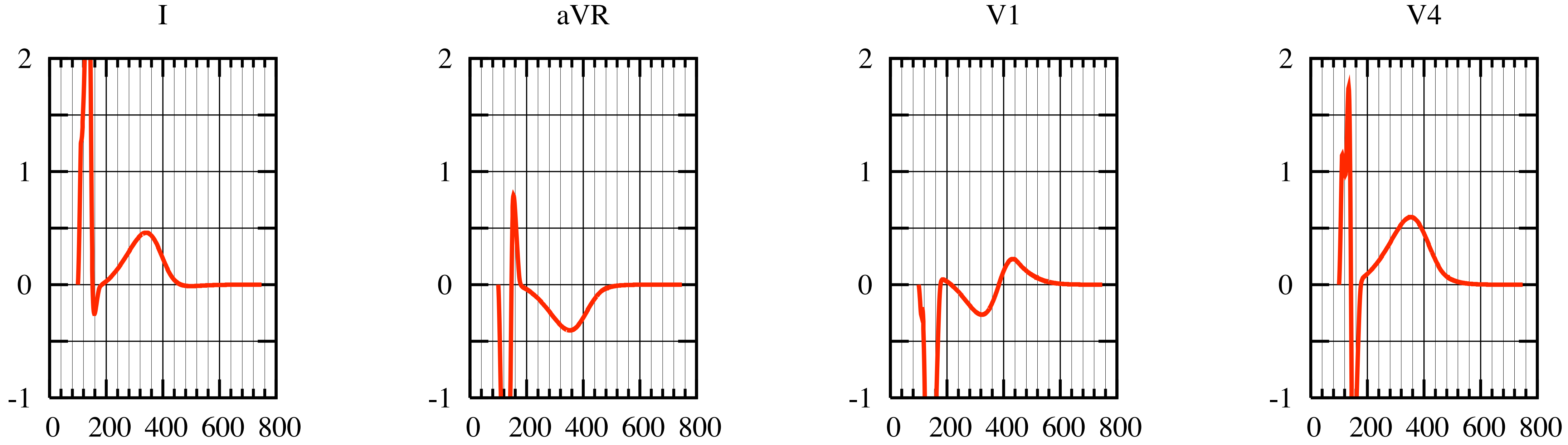
- Impact of heart-torso coupling
- Impact of ionic models
- Cellular heterogeneity

Impact of heart-torso uncoupling

- Full heart-torso coupling:

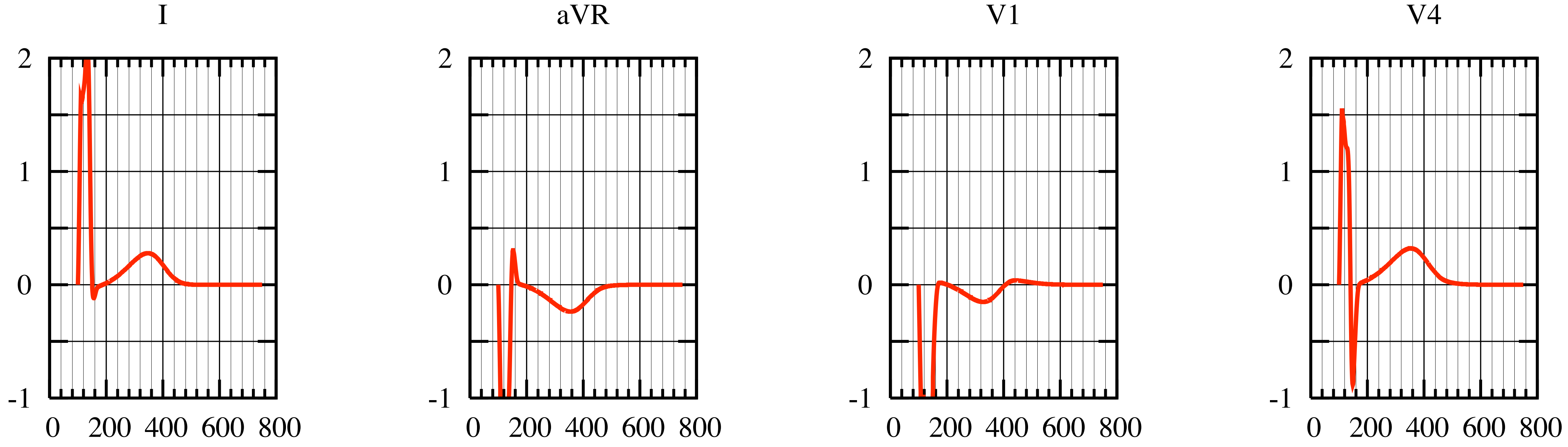


- Uncoupled heart-torso (isolated heart):



Impact of heart-torso uncoupling

- Full heart-torso coupling:



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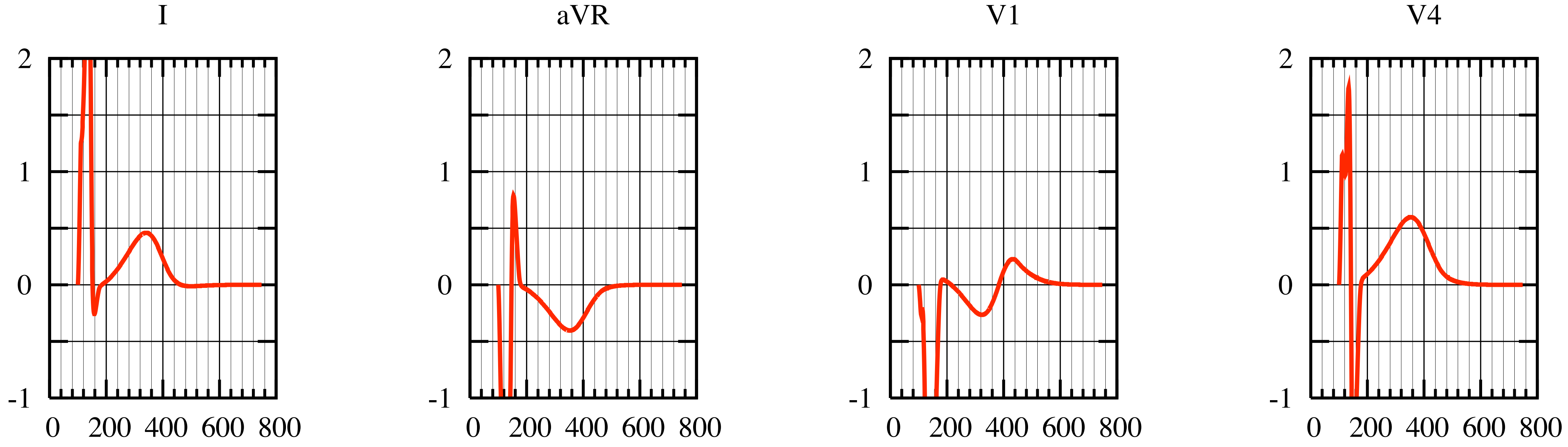
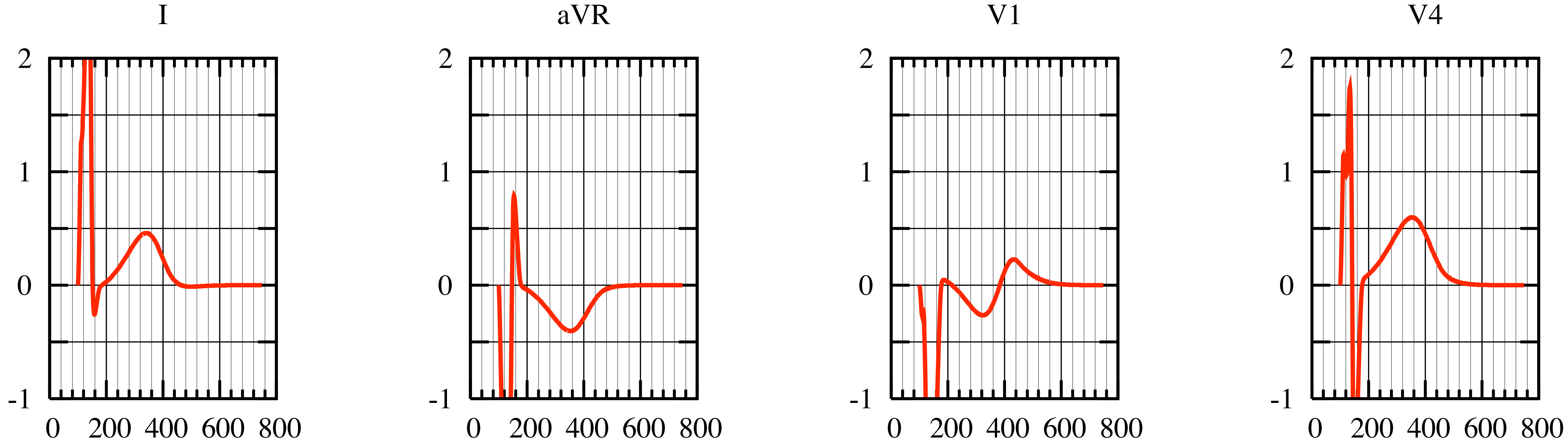


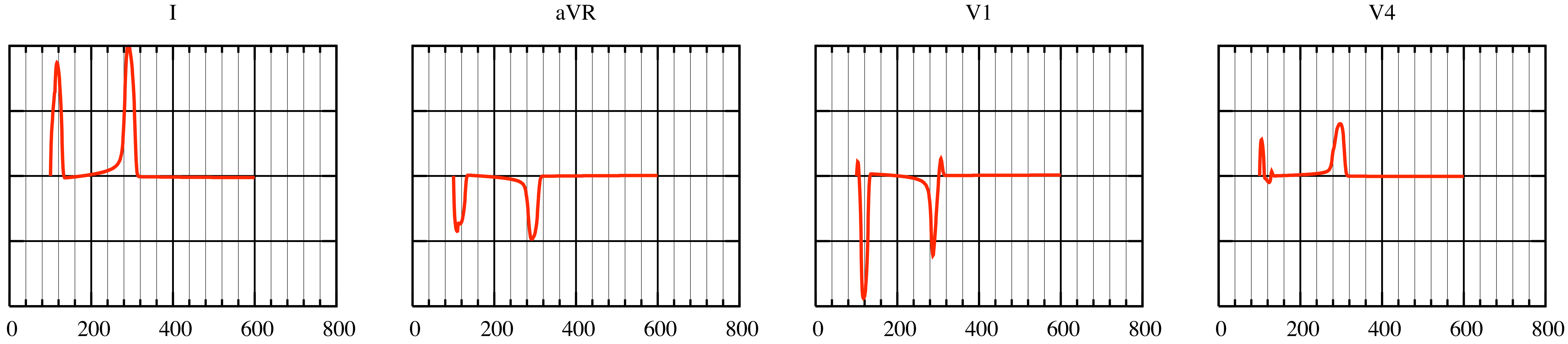
In “agreement” with *Pullan et al. 05*

Impact of ionic model

- Mitchell-Schaeffer:

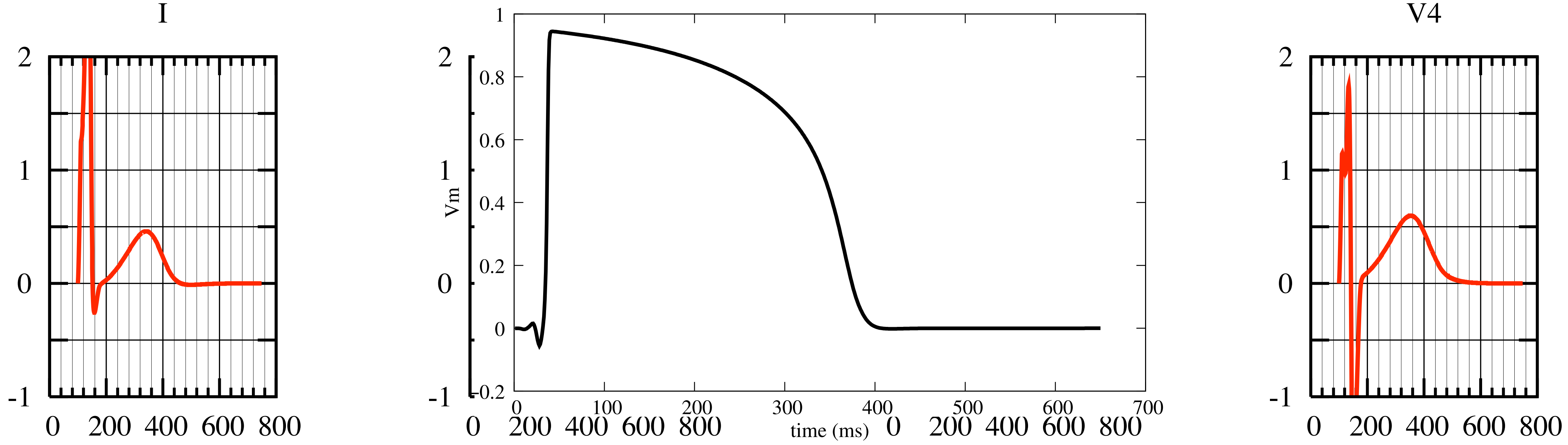


- FitzHugh-Nagumo:

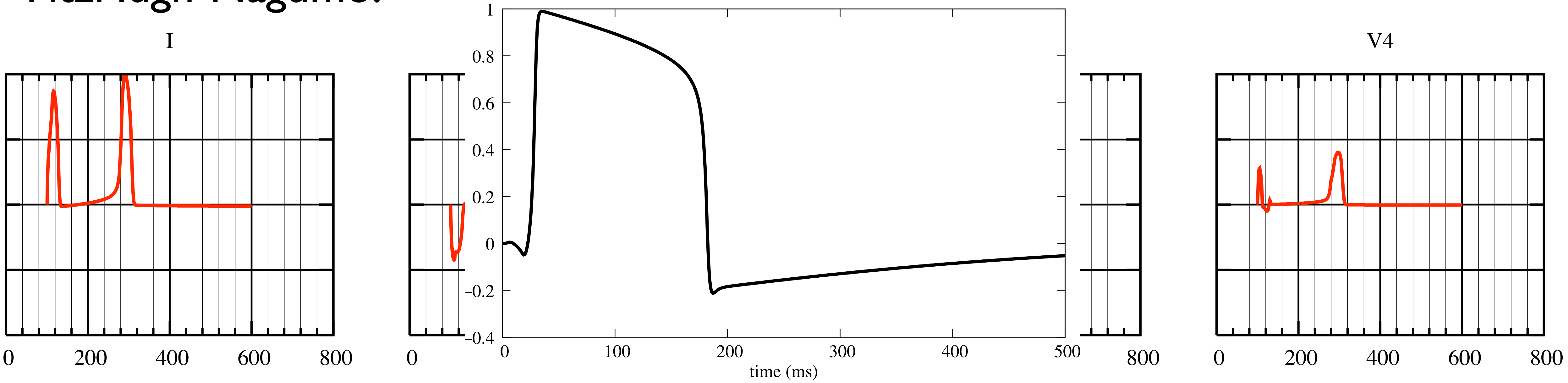


Impact of ionic model

- Mitchell-Schaeffer:



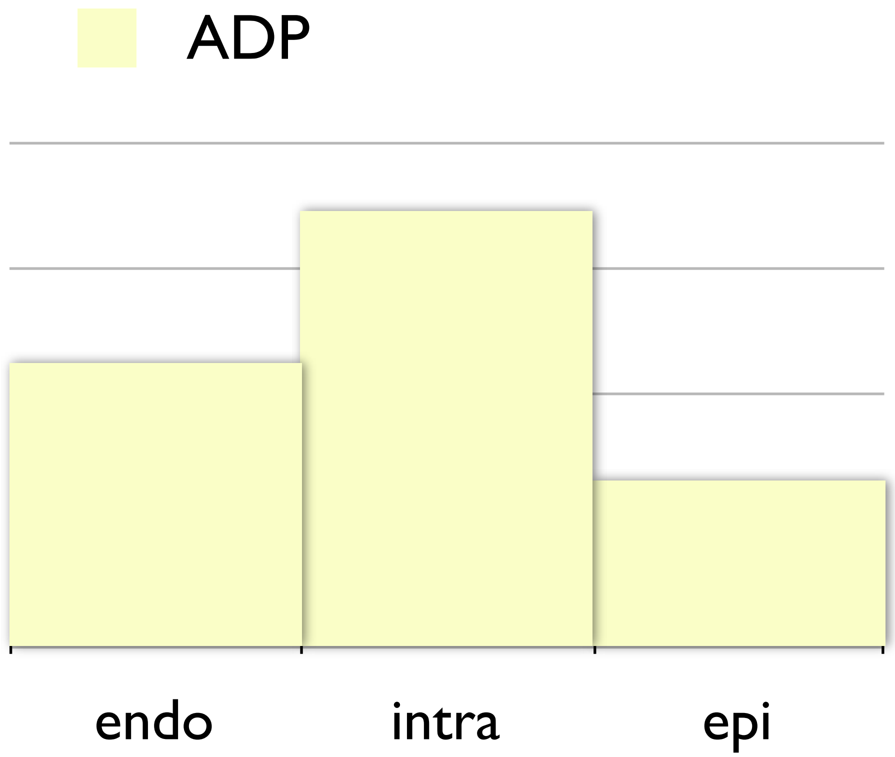
- FitzHugh-Nagumo:



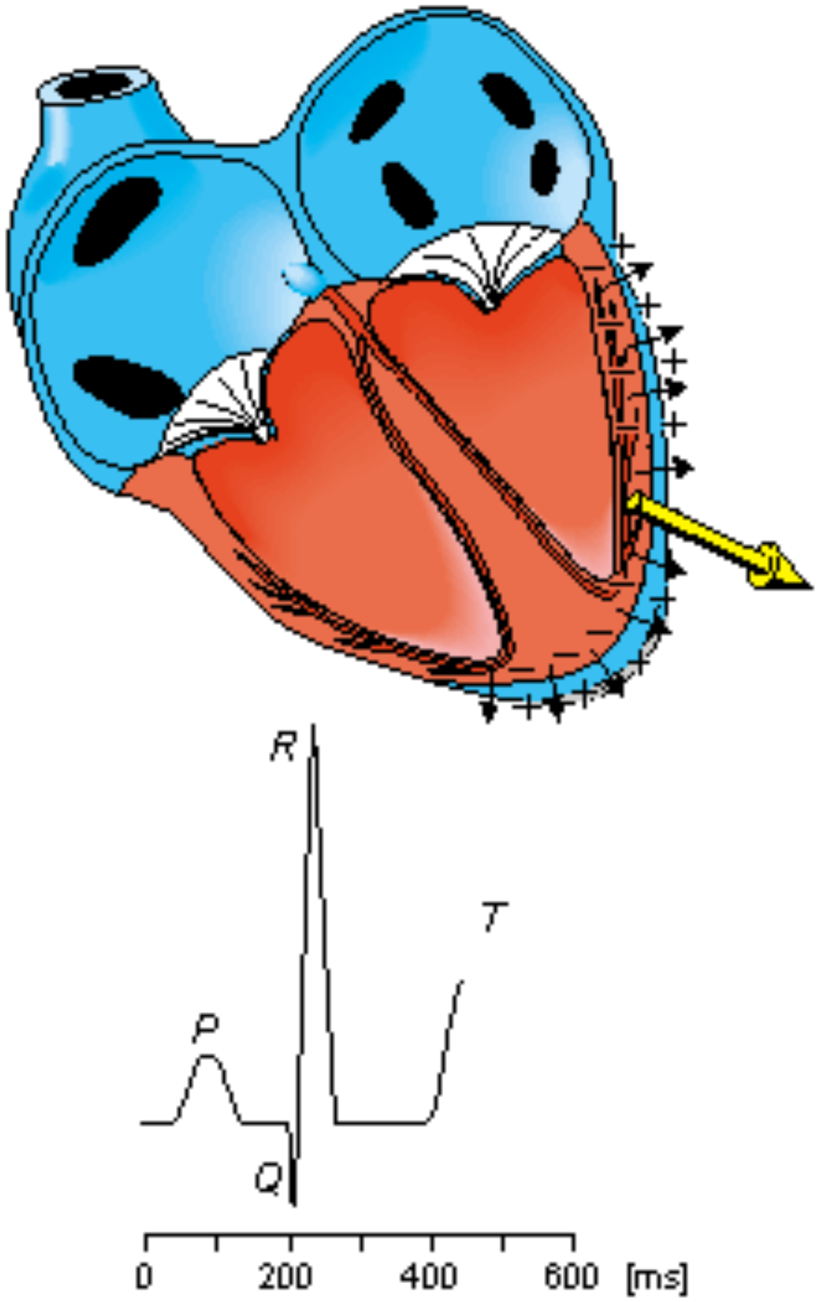
Cell heterogeneity

Cell heterogeneity

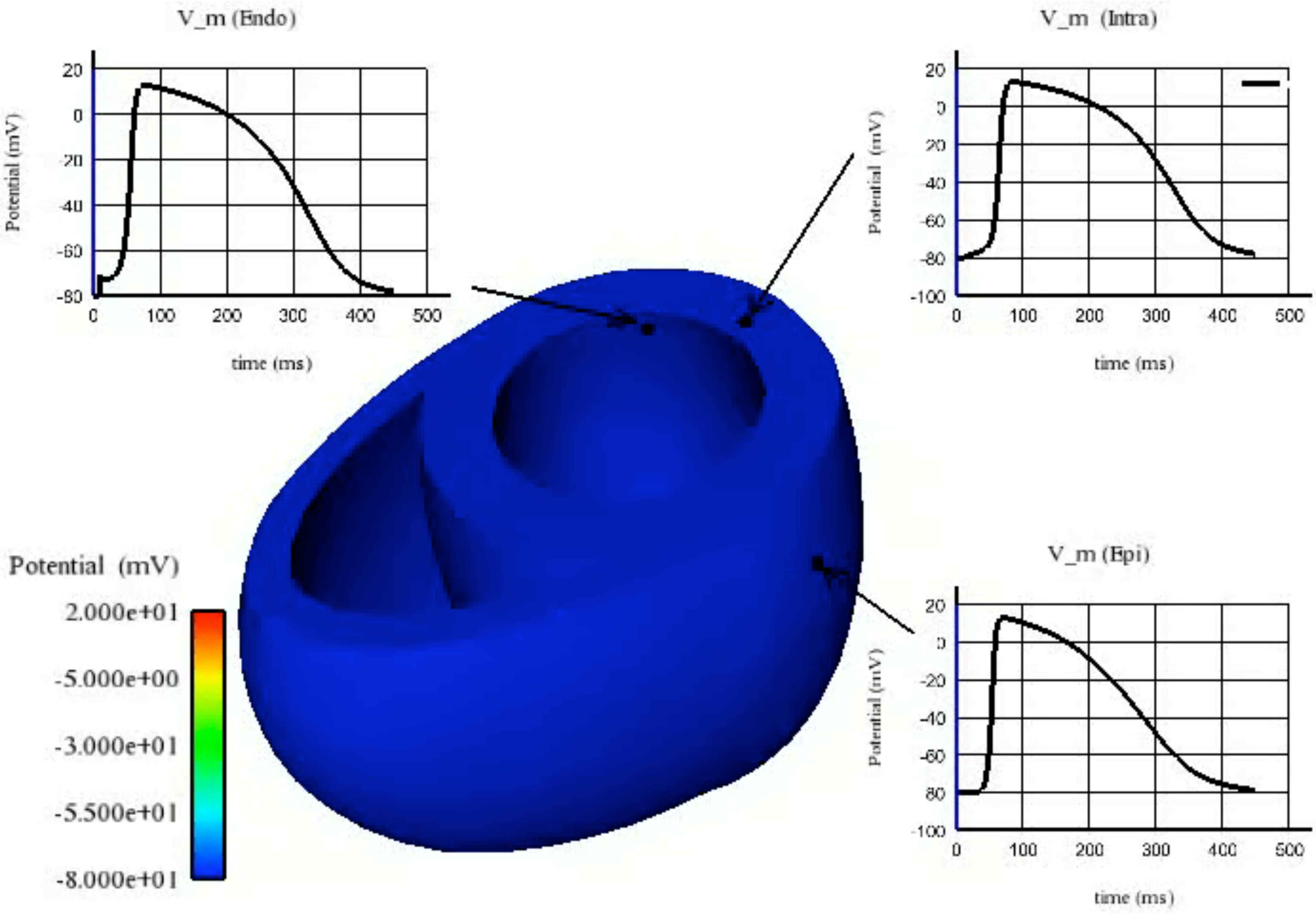
- We cannot consider only one cell type !
- Transmural LV cell heterogeneity



(Drouin et al. 95, Trudel et al. 04, Sampson-Henriquez 05)

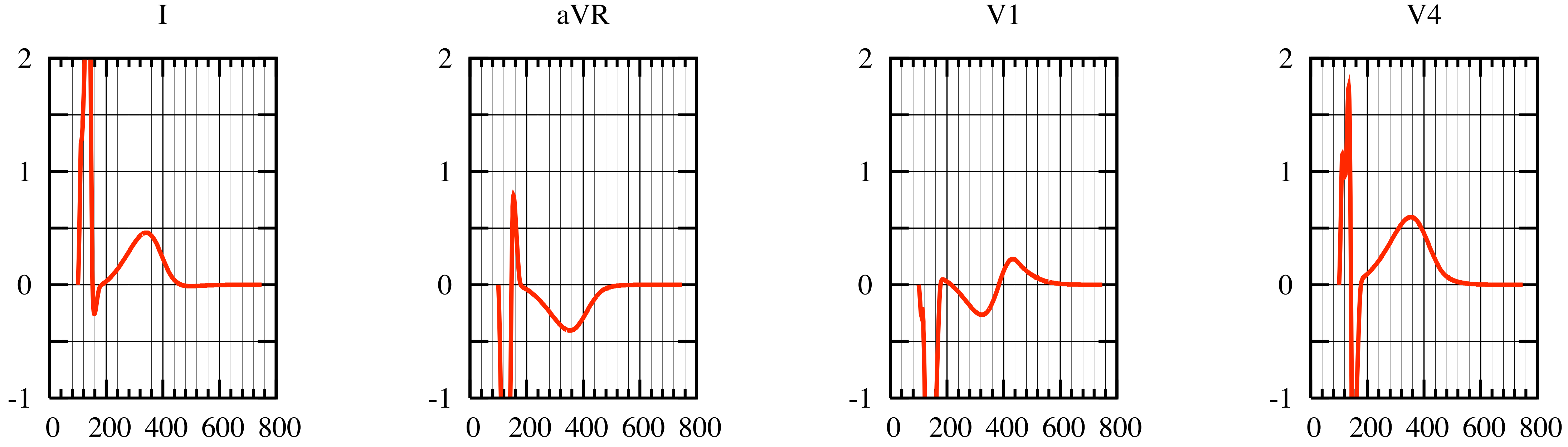


Transmenbrane potential

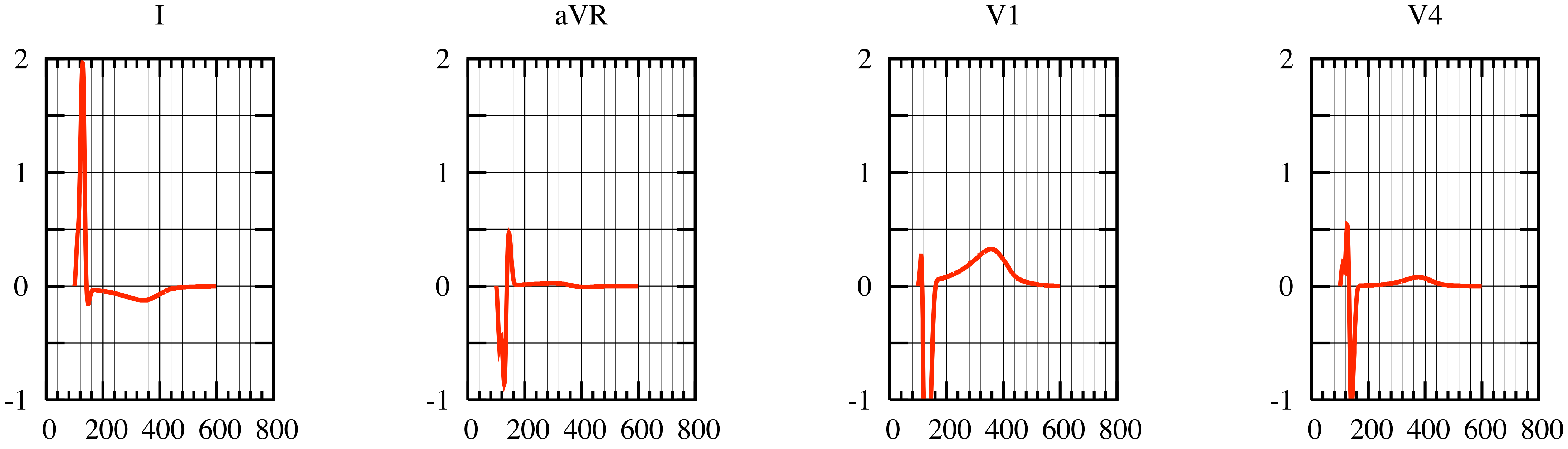


Impact of cell heterogeneity

- Transmural left ventricle APD heterogeneity:



- Homogeneous APD:



Our modelling choices

- Weak coupling with the torso
- Mitchell-Schaefer phenomenological model
- 3 different cells
- Bidomain equations and fibers
- Careful initialization of the stimulation

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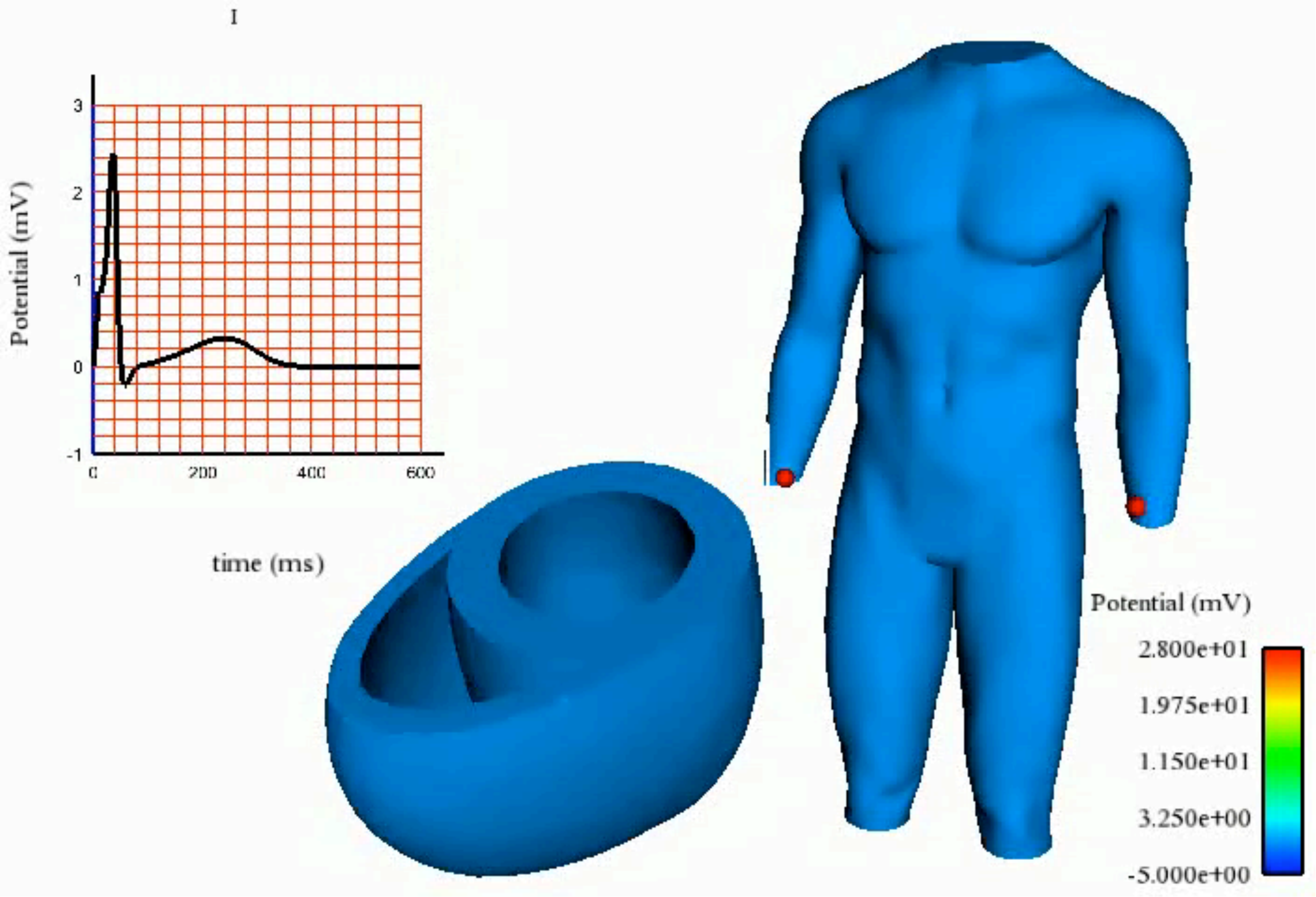
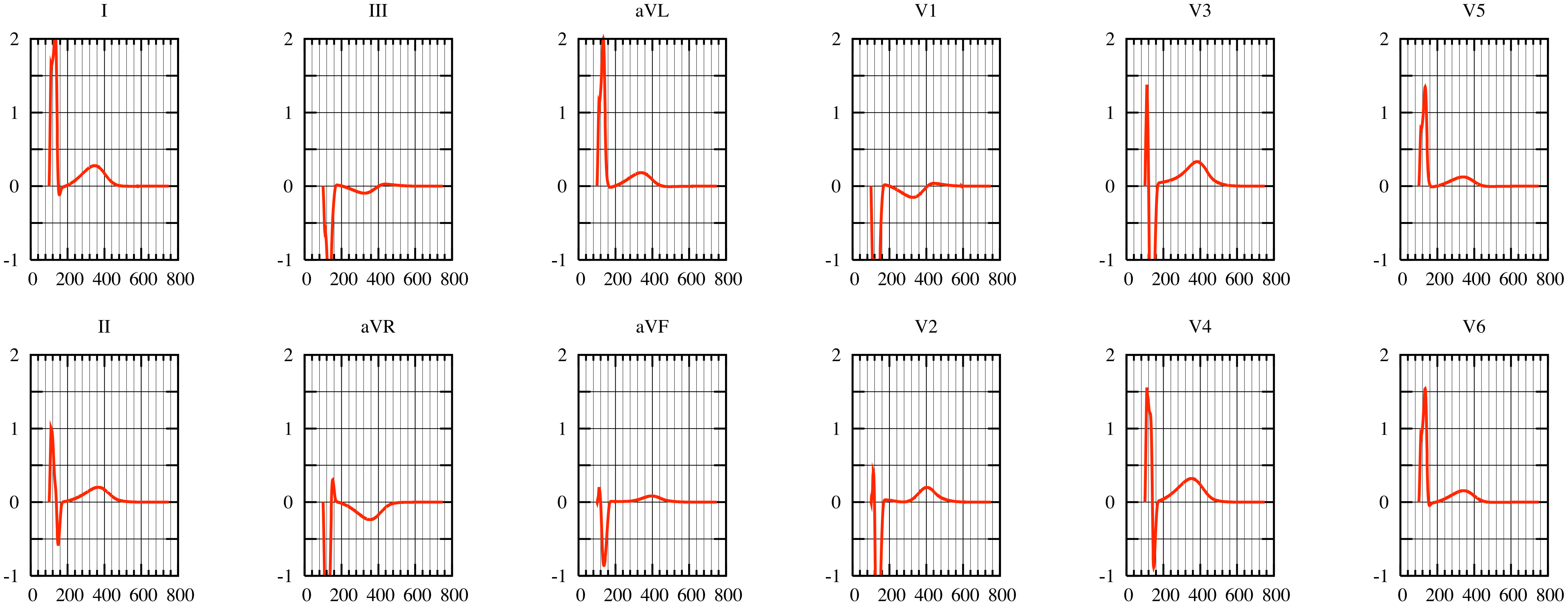
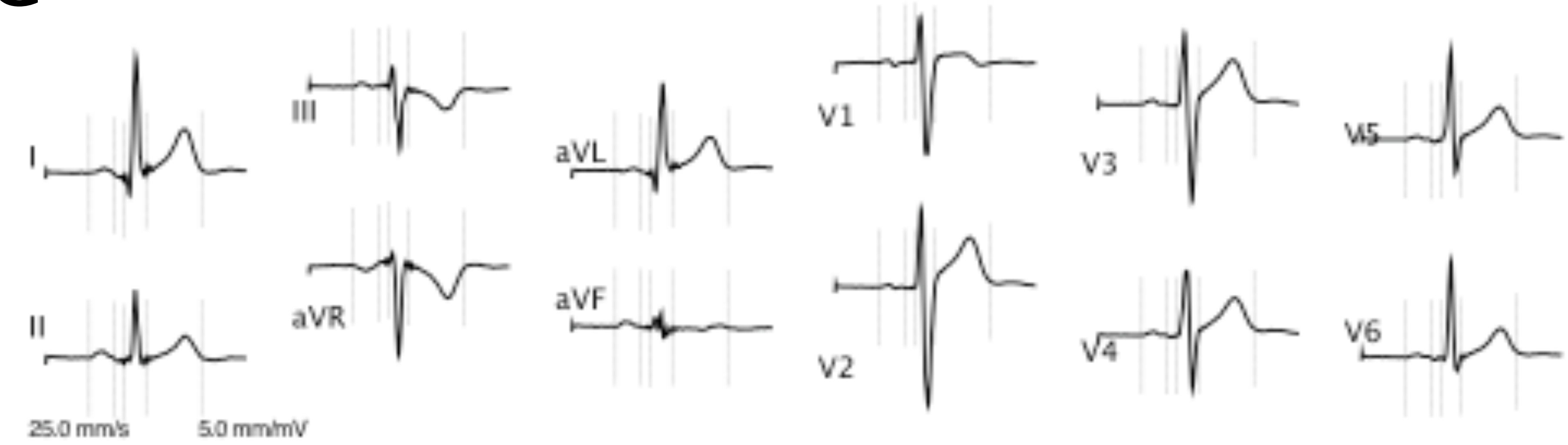


- Simulated ECG



- Real ECG



ECG simulations
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- Difficult to answer... at least it can model a healthy case
- One positive point :

When we simulate a bundle branch block (left or right),
our cardiologist can recognise it on our ECG !

Outline

- Electrical activity of the heart
  - Electrocardiograms (ECG)
  - Cell scale
  - Tissue scale : the bidomain equations
  - ECG simulation
- Applications
  - **Cardiac Resynchronisation Therapy**
  - MRI, Magnetohydrodynamics & ECG

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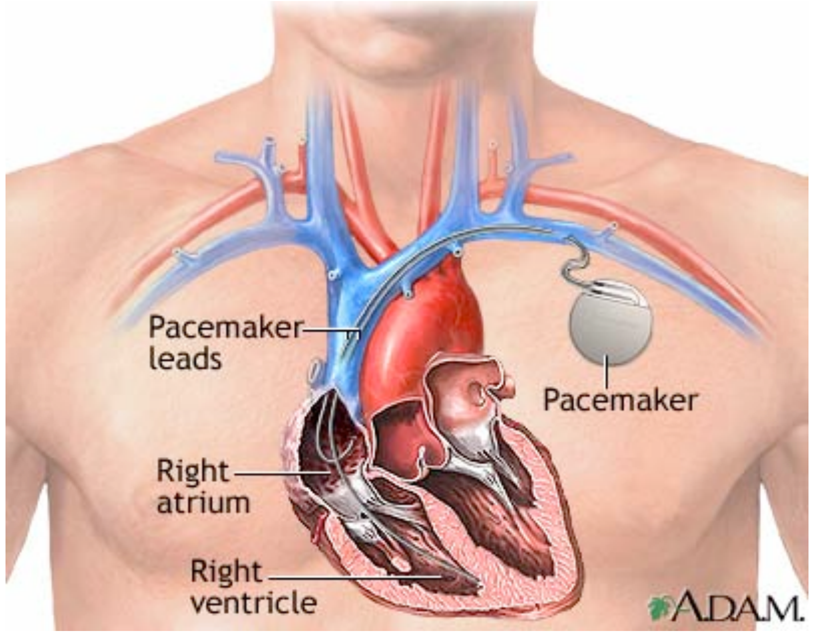

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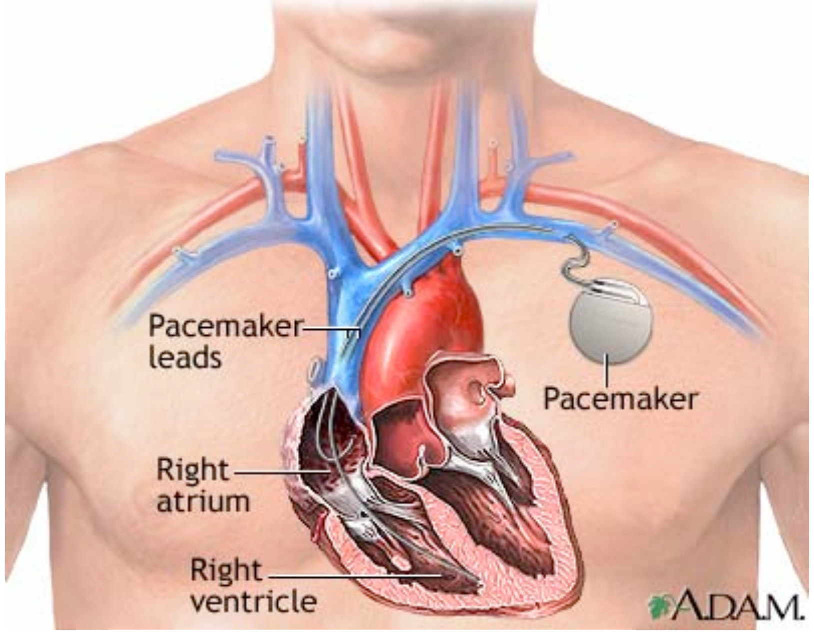
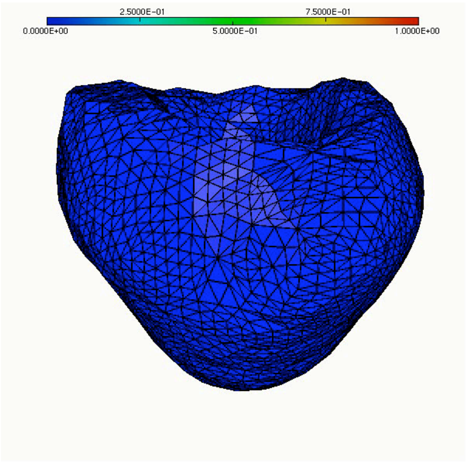


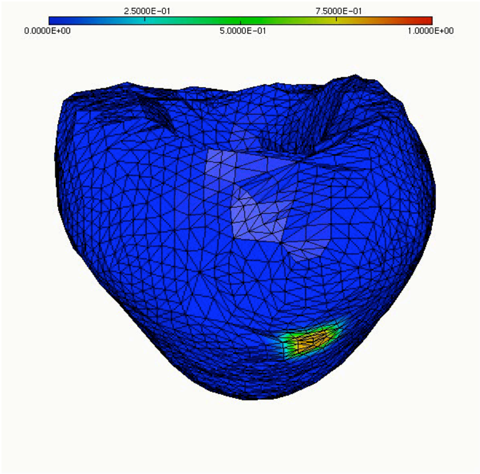
- **Pacing : where ? when ?**

Cardiac Resynchronisation Therapy

Cardiac Resynchronisation Therapy

- Simulation of a reference and a pathological case





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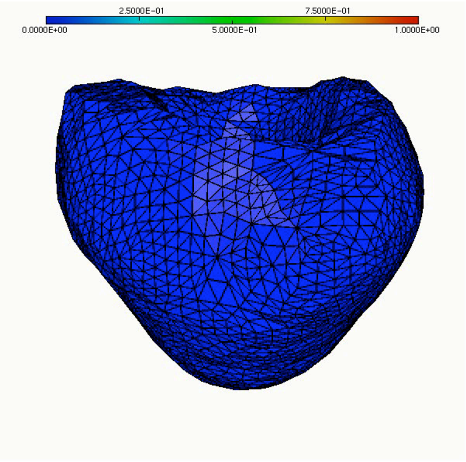

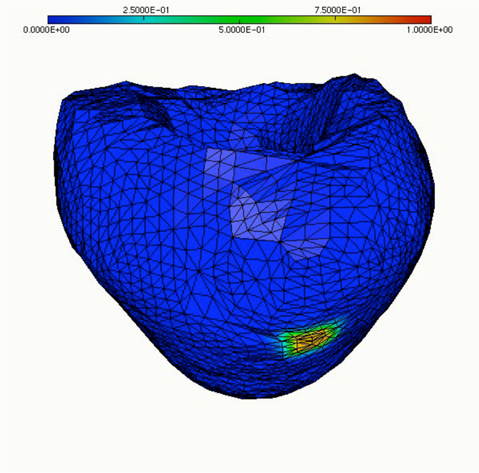

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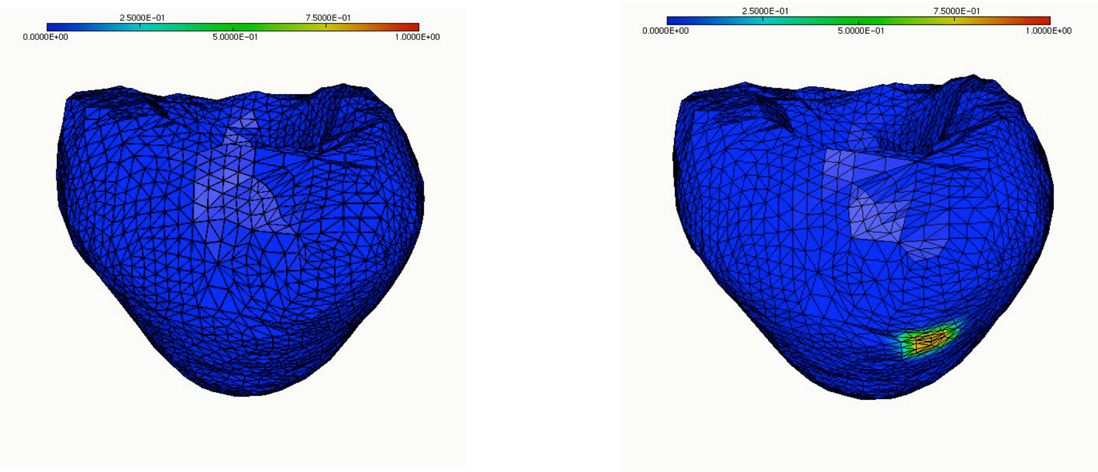

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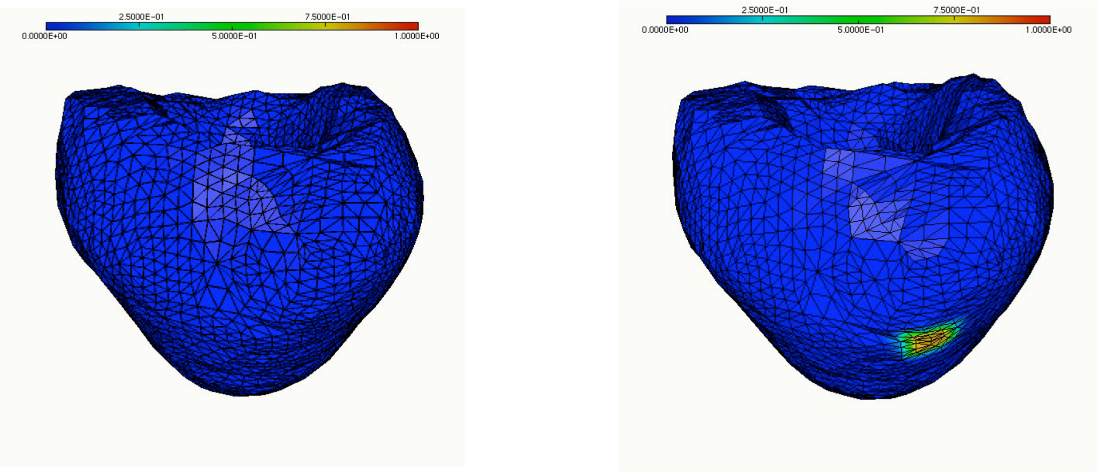

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$$J_1 = |T^*_{\mathrm{ref}} - T^*_{\mathrm{path}}|$$

$$\begin{aligned} T^* &= \inf\{t \geq 0, \quad \mathrm{Volume}(\Omega_t) \geq 0.95\,\mathrm{Volume}(\Omega_H)\} \\ \Omega_t &= \{x \in \Omega_H, \quad V(t,x) > V_s\} \end{aligned}$$

El Alaoui, Dumas, JFG

Optimization with one lead

- Healthy case : $T^*_{\mathrm{ref}} = 28.5\ ms$
- Pathological case : $J_1 = 73\, ms$
- Optimal location: $J_1 = 20.75\ ms$
- Arbitrary location : $J_1 = 65\ ms$

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Other possible choices for the cost function:

- ECG : $$J_2 = \sum_{i=1}^{12} \left(\sum_{n=1}^{N_f} |L^n_{\text{ref,i}} - L^n_{\text{path,i}}|^2 \right)^{\frac{1}{2}},$$
- Mechanical criteria...

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Artifact MHD and MRI

Philips MRI, 3 Teslas

Artifact MHD and MRI

Philips MRI, 3 Teslas

- Permanent uniform magnetic field (typically 1.5 Teslas)
- Today: 3 Teslas (human), 10 Teslas (animals)
- Tomorrow : 10 Teslas (human), 17 Teslas (animals)

Magnetohydrodynamics (MHD)

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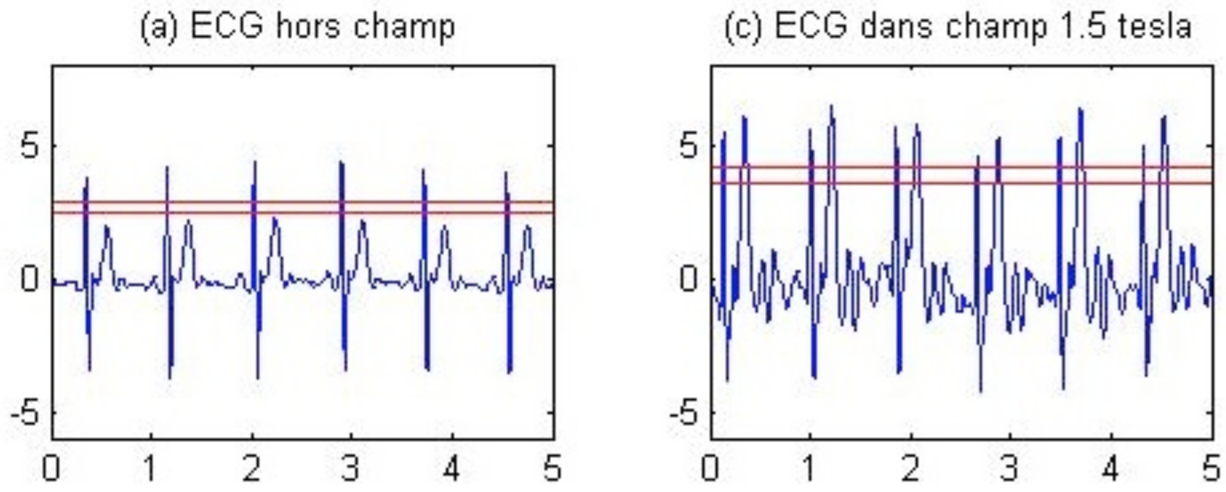

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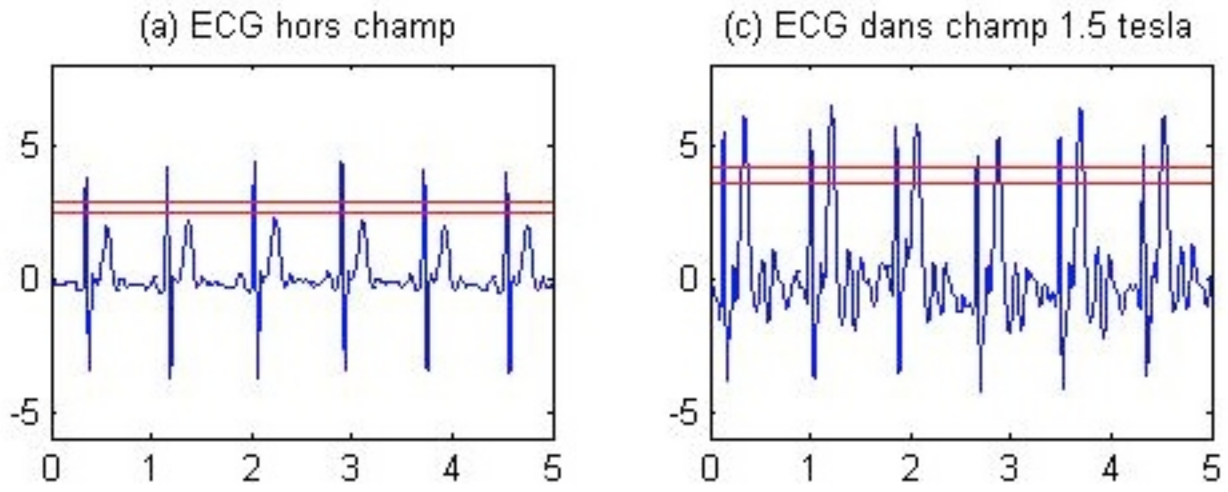

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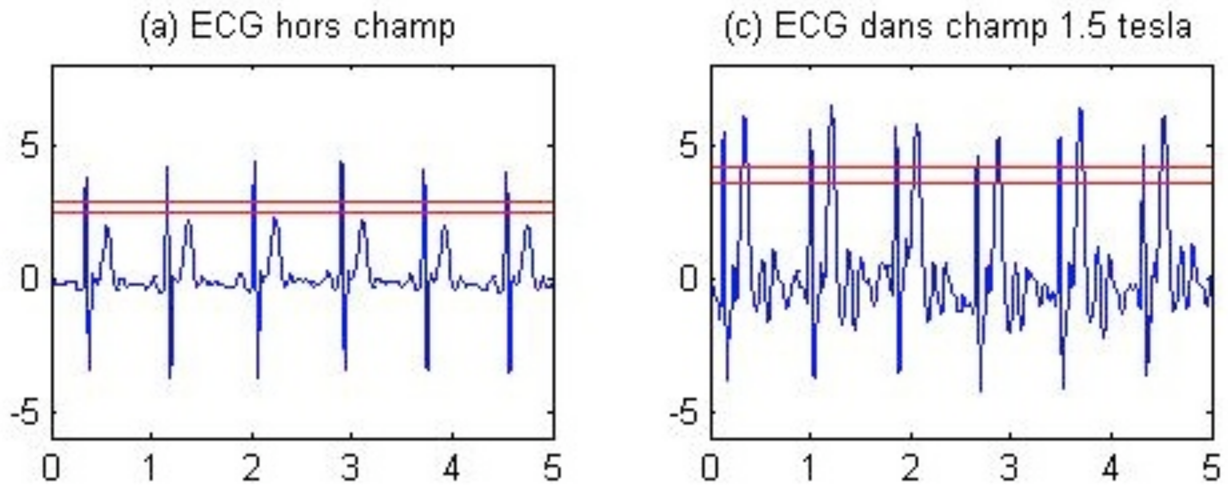

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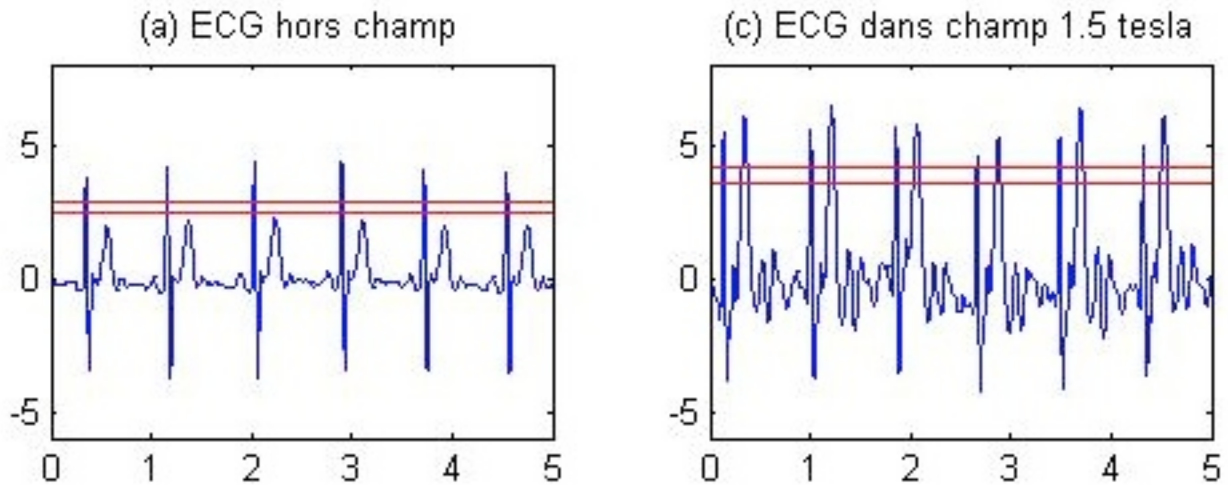

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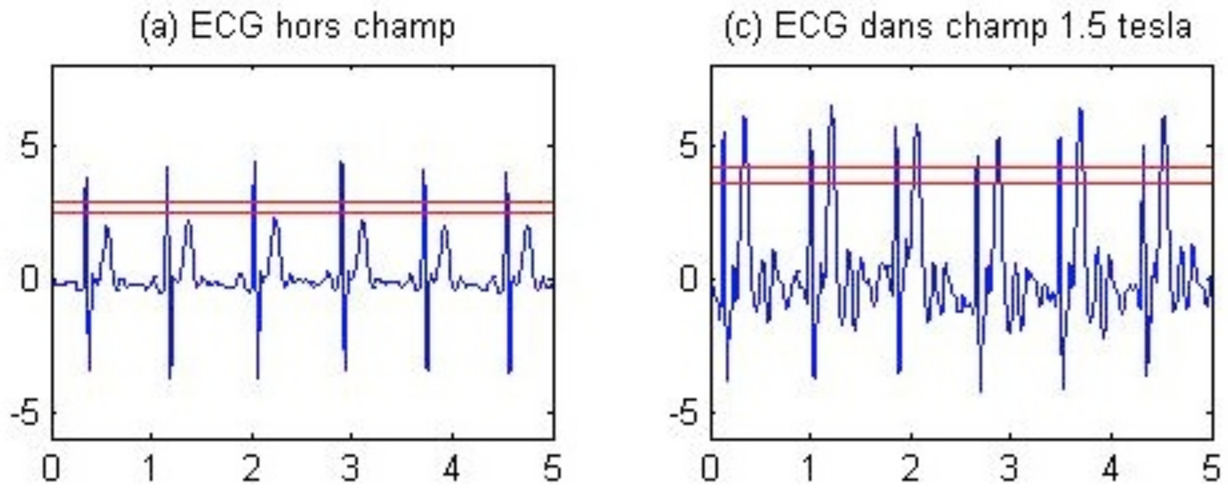

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  - Produce synthetic ECGs to train the signal processing algorithm

MHD and blood flows

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$$\left\{ \begin{array}{rcl} \dfrac{\partial \boldsymbol{u}}{\partial t} + \boldsymbol{u} \cdot \boldsymbol{\nabla} \boldsymbol{u} - \dfrac{1}{Re} \Delta \boldsymbol{u} + \boldsymbol{\nabla} p & = & -\dfrac{Ha^2}{Re} \boldsymbol{\nabla} \Phi_T \times \boldsymbol{B} + \dfrac{Ha^2}{Re} (\boldsymbol{u} \times \boldsymbol{B}) \times \boldsymbol{B}, \\ \mathrm{div}\, \boldsymbol{u} & = & 0, \\ \mathrm{div} \left(\dfrac{\sigma}{\sigma_0} \boldsymbol{\nabla} \Phi_T \right) & = & \mathrm{div} \left(\dfrac{\sigma}{\sigma_0} \boldsymbol{u} \times \boldsymbol{B} \right) \end{array} \right.$$

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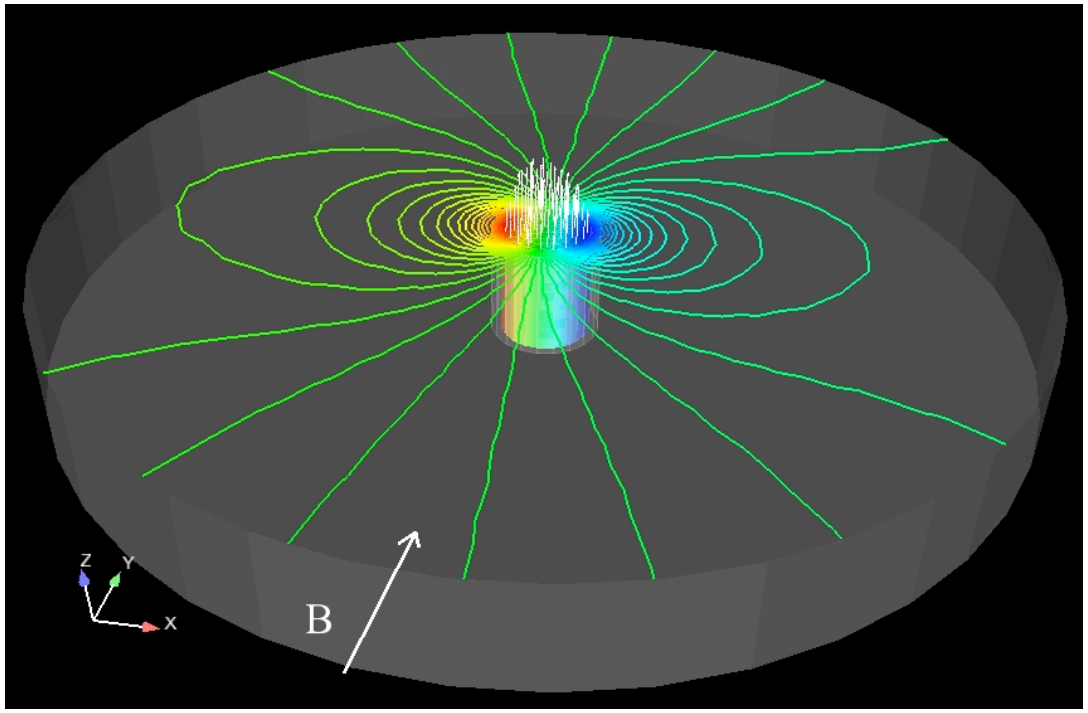

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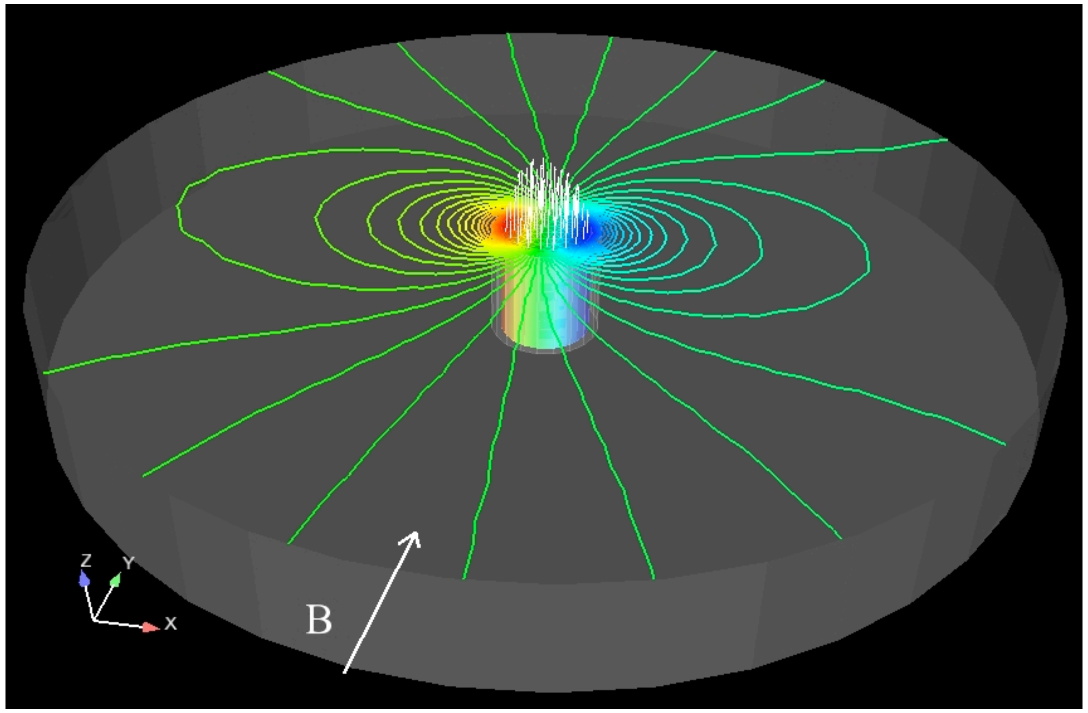

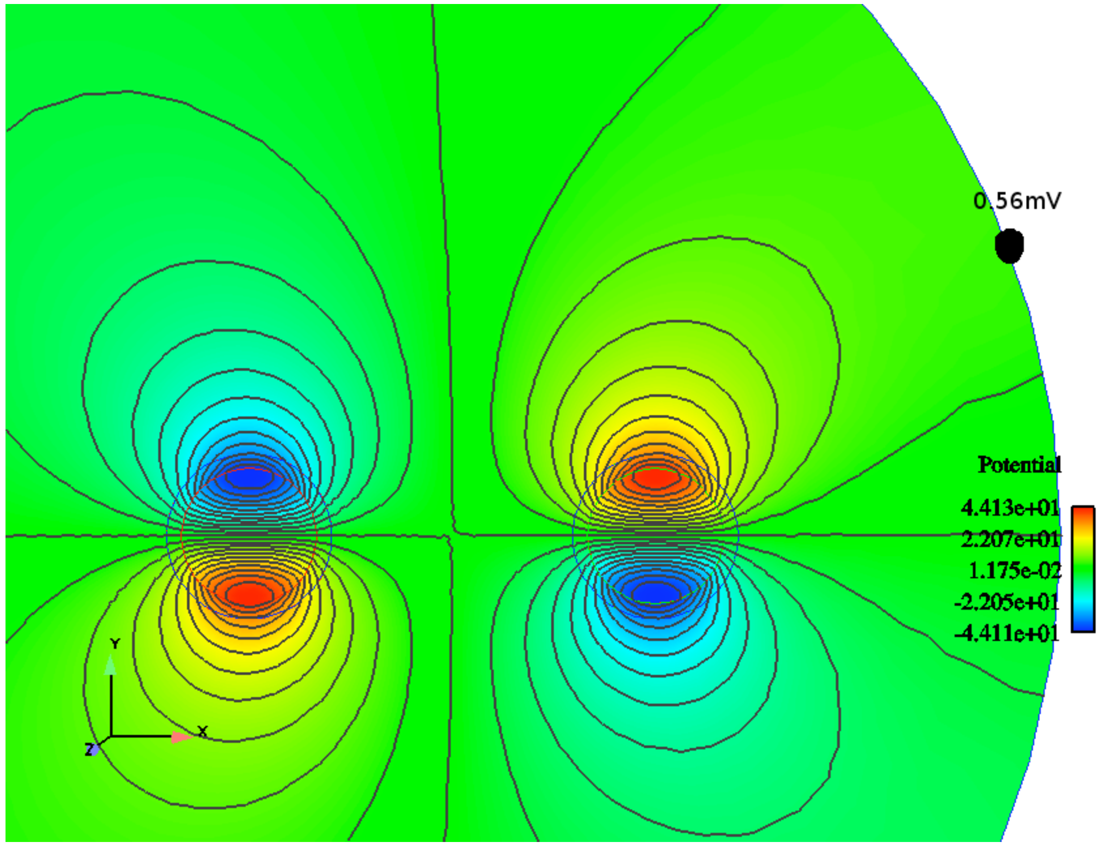

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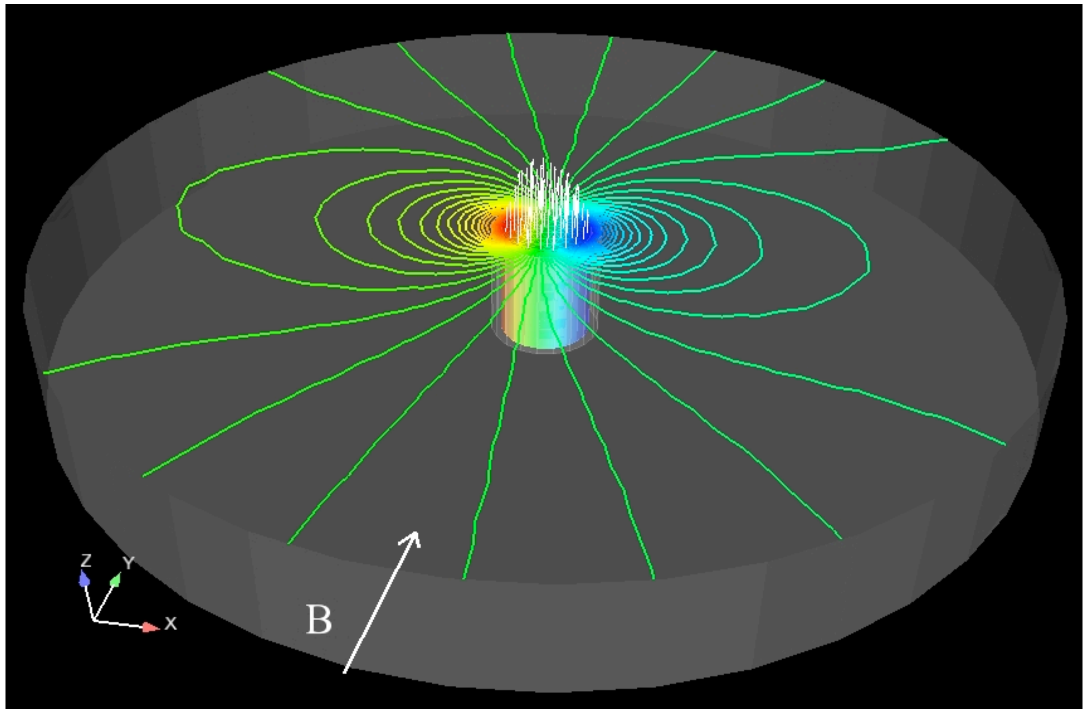
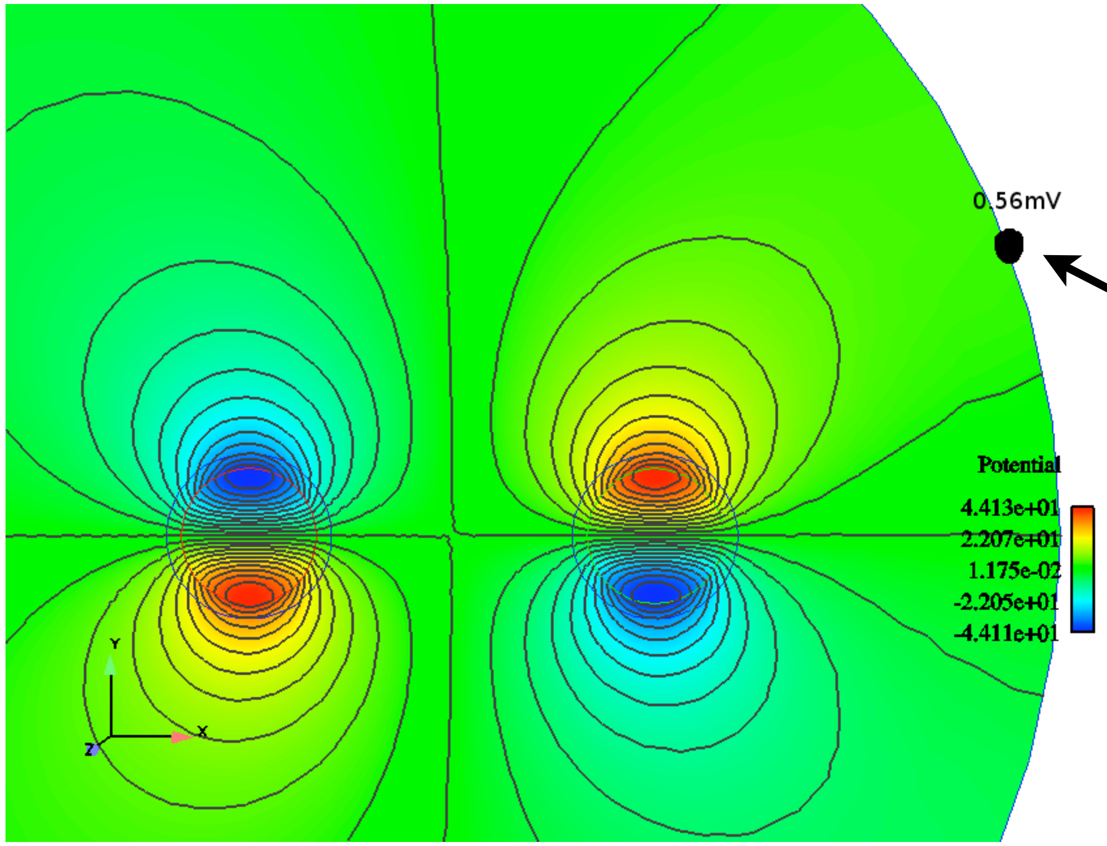
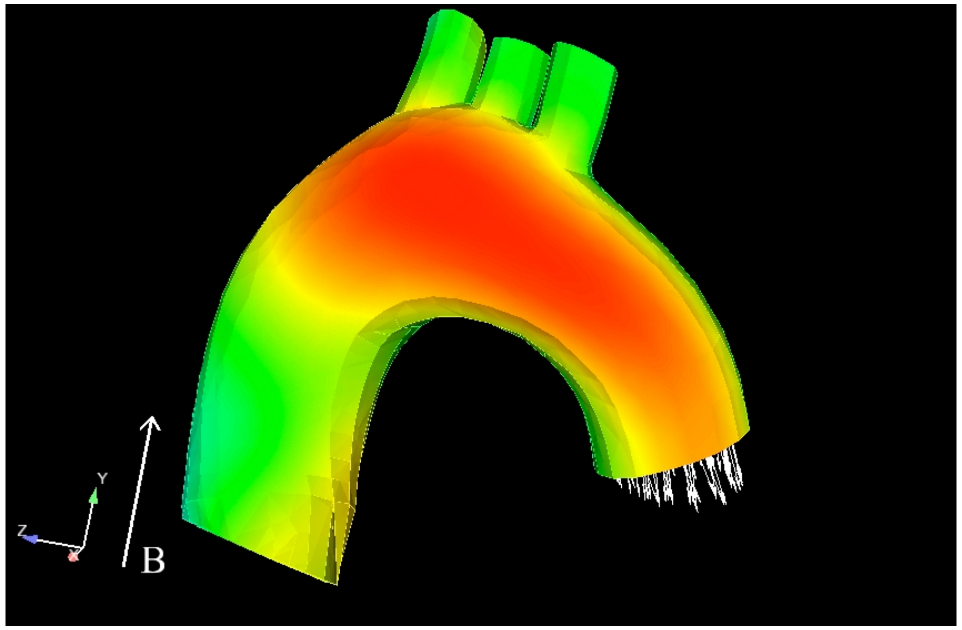


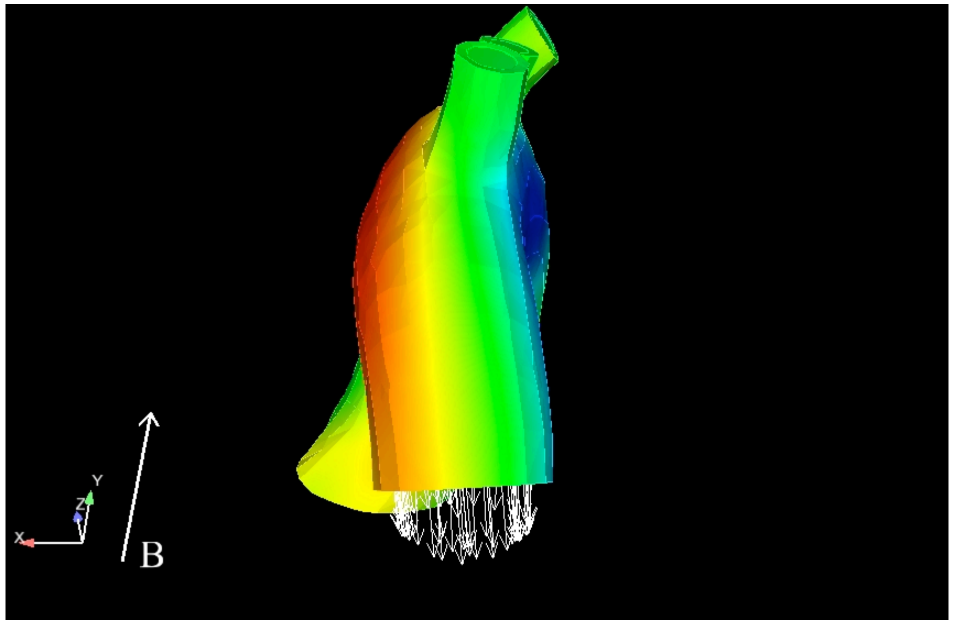


2D value = 0.57 mV

MHD in blood flows

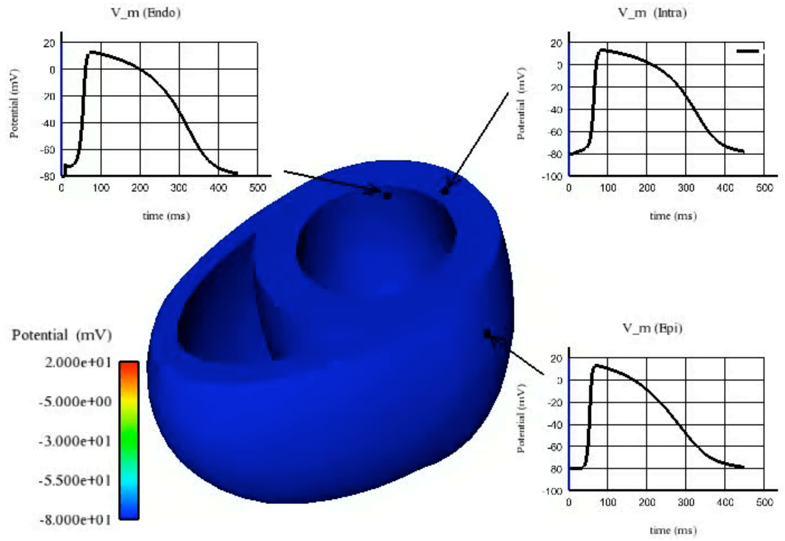
- Preliminary results:





JFG-Martin

MRI & MHD : summary



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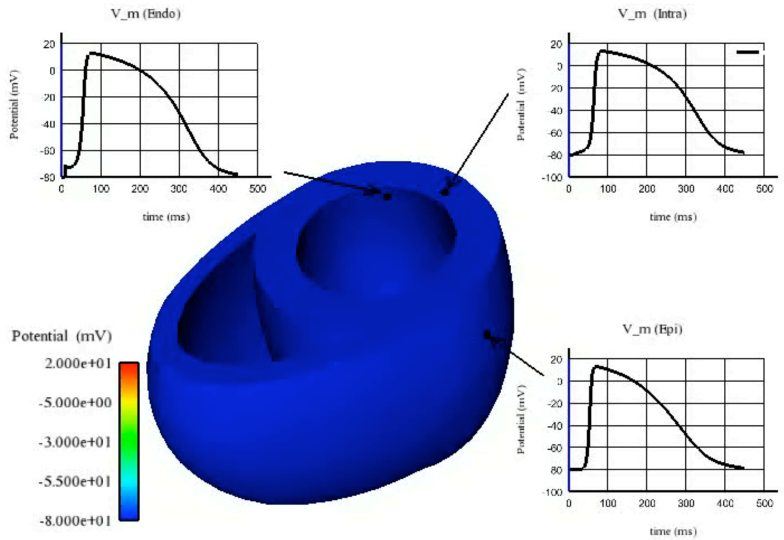

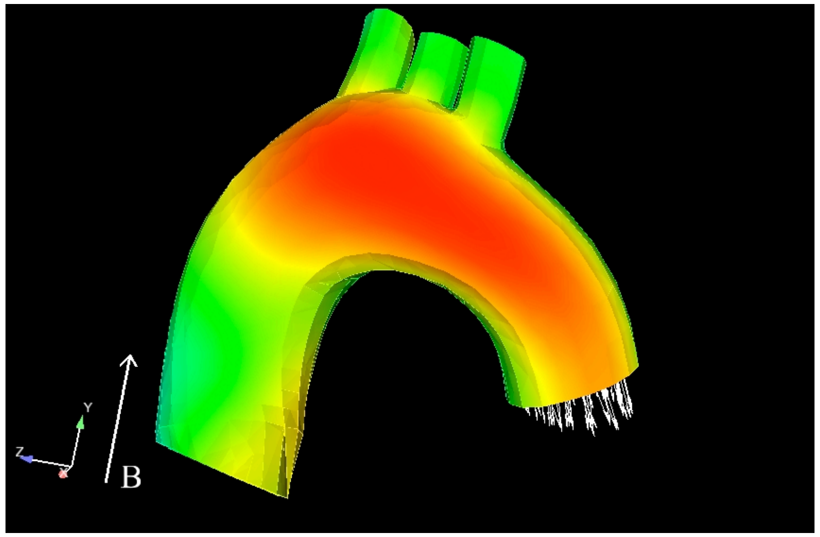

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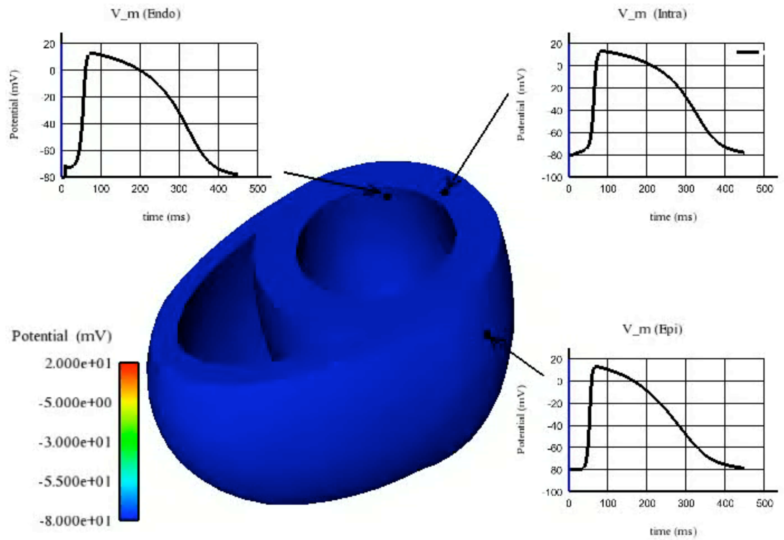
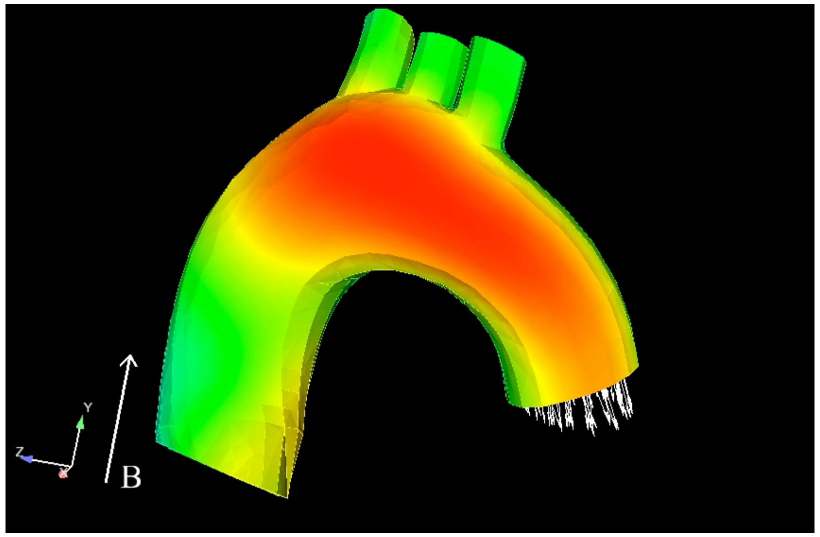
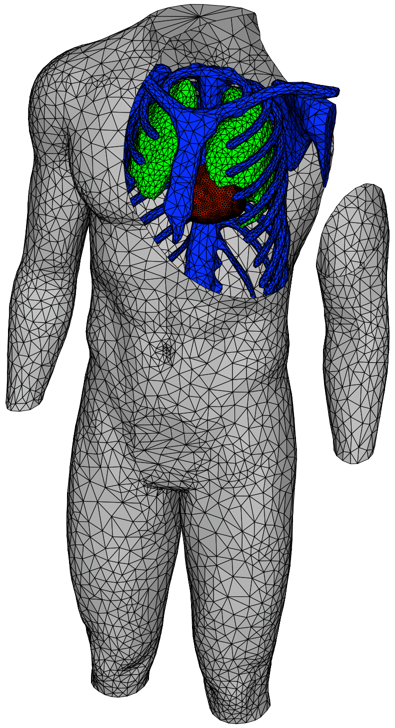
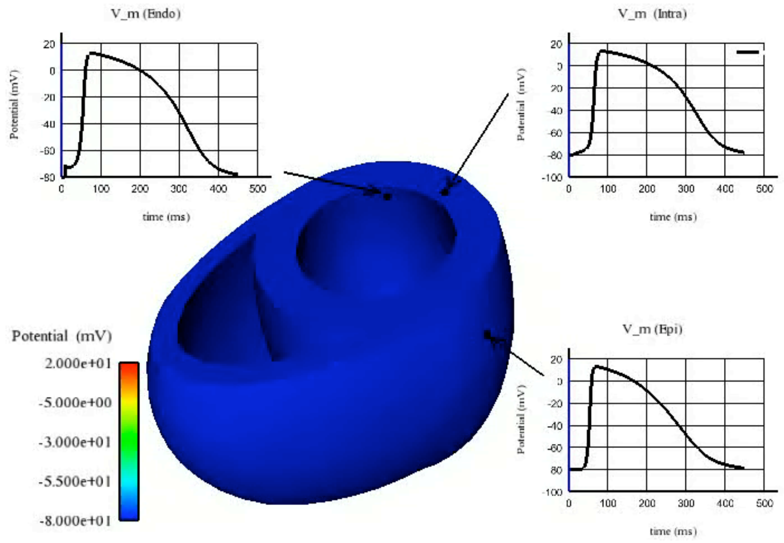
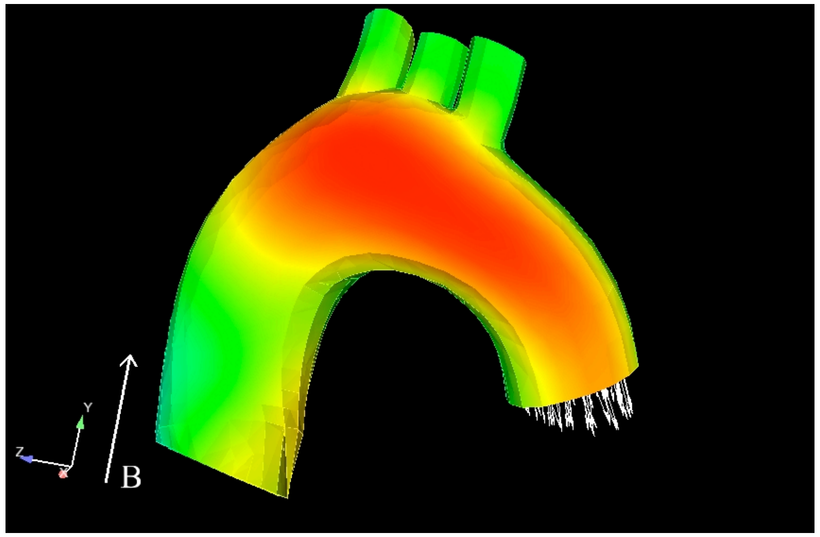


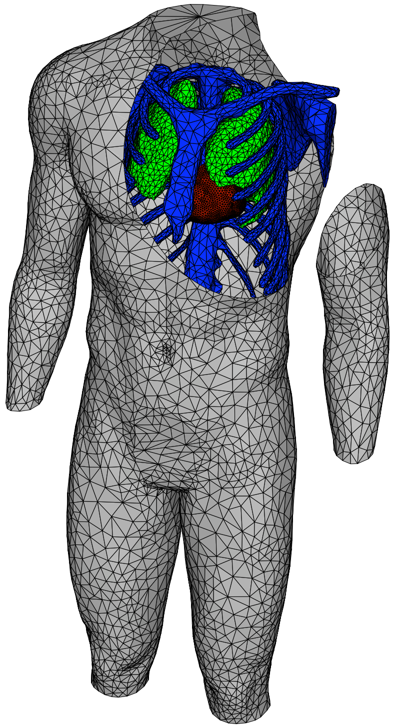


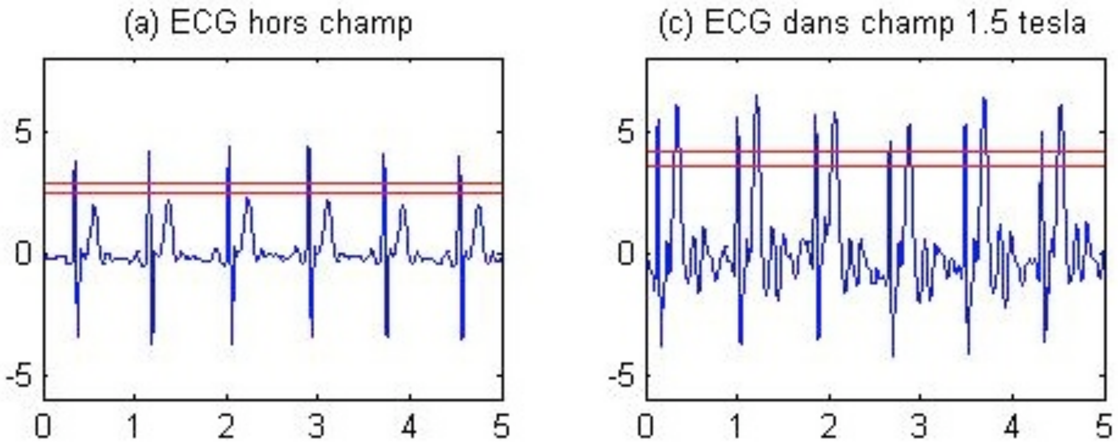


MRI & MHD : summary









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The REO team, june 2007 (INRIA Paris-Rocquencourt / Univ. Paris 6)