Regulatory and economic capital requirements related to credit risk

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2 Regulatory Capital
- Standard approach
- IRB approach

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- Vasicek approach for default rates
- Stochastic LGD
- Correlation between portfolios

4 Proposed problem for the study week

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Basel II framework

The second of the Basel Accords[1] **(Basel II)** defines the framework to ensure that the more significant the risk a bank is exposed to, the greater the amount of capital the bank needs to hold to safeguard its solvency and overall economic stability. Two main concepts of Basel II framework are:

- **Regulatory Capital:** Regulatory requirements that determine how much liquid capital the banks must keep on hand, concerning their overall holdings. The calculation of regulatory capital is usually based on arbitrary formulas and considerations defined by regulation.

- **Economic Capital:** Pillar II of Basel II defines an exercise of self-assessment of capital in which, taking into account its risk profile and the current economic and financial environments, banks should identify all the material risks that affect the institution and evaluate them comprehensively, concluding on capital adequacy.

[1] After the financial crisis (2008-09), this reform was updated into Basel III Accords, without important changes on the topics discussed in this presentation.

Definitions

To estimate **economic capital**, models split the loss distribution into two segments:

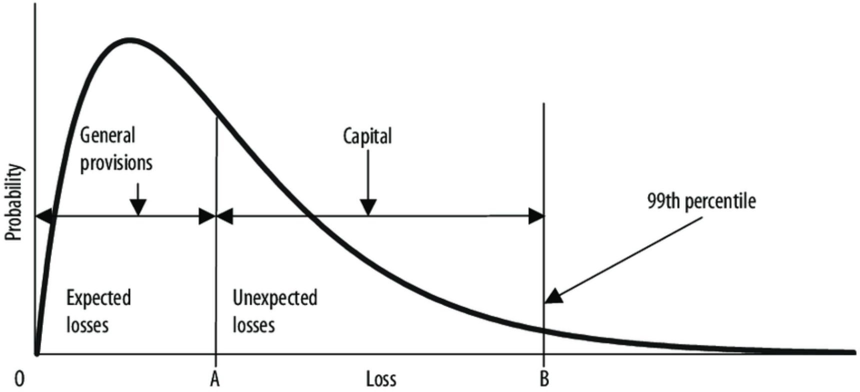
- **Expected Loss** ("likely"): estimated as the mean of the losses.
- **Unexpected Loss** ("unlikely"): estimated by setting an extremely high threshold (unlikely probability)

The difference between unexpected and expected loss serves as an estimate for economic capital.

Moreover, it is assumed that:

- reserves and provisions held on the balance sheet of the bank should adequately cover and compensate for "expected" loss caused by normal operating conditions
- economic capital should cover the "unexpected" portion of the loss distribution

Loss distribution



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Standard approach

The **standard approach** is the simplest way to compute regulatory capital for credit risk and usually the one that gives the **highest capital requirements**, because of its lack of sensitivity.

The standard approach relies in the following key elements:

- Categorization by exposure type as defined by Basel II accord.
- Apply risk weights per exposure as defined by Basel II accord and compute total risk weighted assets (RWA).

The regulatory capital for credit risk is then calculated as **8 % of the total RWA** under Basel II.

IRB approach

Under the Basel II guidelines, banks are allowed to use their own estimated risk parameters for calculating regulatory capital. This is known as the **internal ratings-based (IRB**) approach for capital requirements for credit risk.

In a simplified way, IRB approach relies in the following key elements:

- **Classification by exposure type** as defined by the Basel II accord
- For each exposure class, banks should estimate the risk parameters:
  - **Probability of default (PD)**: Likelihood of the default event
  - **Loss given default (LGD)**: Percentage of loss that the bank does not recover after the default
  - **Exposure at default (EAD)**: Loss at the moment of default
- **Risk-weight formula**, which provides risk weights. It is a regulatory closed formula, based on the Vasicek approach (arts. 153-154 CRR[2]), in which PD, LGD, EAD and M (maturity, explicit component) are used as inputs.

Then, the regulatory capital for credit risk is the **8 % of the total RWA**.

[2] Capital Requirements Regulation (EU) No. 575/2013

IRB: Risk-weight formula

- As defined in art. 153 of CRR, for exposures to corporates, institutions and central governments and central banks the risk-weight formula is:

$$RW = \left(LGD \cdot \left(\frac{1}{\sqrt{1-R}} \cdot G(PD) + \sqrt{\frac{R}{1-R}} \cdot G(0,999) \right) - LGD \cdot PD \right) \cdot \frac{1 + (M - 2,5) \cdot b}{1 - 1,5 \cdot b} \cdot 12,5 \cdot 1,06 \tag{1}$$

- As defined in art. 154 of CRR, for retail exposures the risk-weight formula is:

$$RW = \left(LGD \cdot \left(\frac{1}{\sqrt{1-R}} \cdot G(PD) + \sqrt{\frac{R}{1-R}} \cdot G(0,999) \right) - LGD \cdot PD \right) \cdot 12,5 \cdot 1,06 \tag{2}$$

where

- $N(x)$: the cumulative distribution function of a standard normal random variable
- $G(Z)$: the inverse cumulative distribution function of a standard normal random variable
- R: the correlation coefficient

The difference between equations (1) and (2) is known as the **maturity adjustment**. We will not take into account this adjustiment in the proposed problem.

IRB: Correlation coefficient R

The **correlation coefficient** is defined for exposures to corporates, institutions and central governments and central banks as

$$R = 0,12 \cdot \frac{1 - e^{-50 \cdot PD}}{1 - e^{-50}} + 0,24 \cdot \left(1 - \frac{1 - e^{-50 \cdot PD}}{1 - e^{-50}}\right) \tag{3}$$

and for retail exposures as

$$R = 0,03 \cdot \frac{1 - e^{-35 \cdot PD}}{1 - e^{-35}} + 0,16 \cdot \left(1 - \frac{1 - e^{-35 \cdot PD}}{1 - e^{-35}}\right) \tag{4}$$

Foundation and Advanced IRB

Foundation IRB (F-IRB)

- Empirical model to estimate the PD (probability of default) for individual clients or groups of clients
- LGD is given by regulation
- Exposure is deterministic, as in the standard approach

Advanced IRB (A-IRB)

- PD (probability of default) is estimated using historical data
- LGD is estimated using historical data
- Exposure is estimated using historical data (it is not deterministic!)

Unfortunately, it's not that simple...

Other IRB relevant topics:

- Definition of default
- Quantitative and qualitative risk discrimination
- Parameter adjustments for PD and LGD: Point-in-time, long-run, through-the-cycle, downturn, ...
- Limited amount of closed recovery processess for LGD
- Representativeness of data
- Backtesting
- Collaterals, contract reestructuring, special treatments, ...

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Value at risk

Banks develop **economic capital models for credit risk** that allow them to establish profitability objectives based on the capital assigned to each area or product, making management decisions and overcoming the limitations of the Basel models of regulatory capital.

A certain amount of capital reserve is required as a cushion for potential large losses in the portfolio. The most popular risk measure for this purpose is the **Value at Risk (VaR)**, which is defined as follows:

If α is some confidence level, the VaR_α is simply the α-quantile of the loss distribution of L. Thus,

$$VaR_\alpha = \text{ínf}\left\{x : P(L \leq x) \geq \alpha\right\} \tag{5}$$

We choose $\alpha = 99,9$ as confidence level to be consistent with the regulatory capital premises, and one year time horizon.

Portfolio loss

We consider a **credit portfolio** consisting of n $(n \to \infty)$ homogeneous obligors or counterparties with exposures $E_i, i = 1, \ldots, n$.
We assume that obligor i defaults if its standardized log asset value X_i is less than some default threshold y_i (related to debt level) after a fixed time horizon.
The **event of default** can be modelled as a Bernoulli random variable $D_i = 1_{\{X_i < y_i\}}$ with known default probability $PD_i = P(X_i < y_i)$ as follows:

$$\begin{cases} P(D_i = 1) = PD_i \\ P(D_i = 0) = 1 - PD_i \end{cases} \tag{6}$$

If we assume that some fraction (R_i) can be recovered in case of default of obligor i, it follows that the loss L_i due to obligor i default is simply $(1 - R_i) \cdot E_i \cdot D_i$, so that the **portfolio loss** is given by

$$L = \sum_{i=1}^{n} L_i = \sum_{i=1}^{n} E_i \cdot D_i \cdot (1 - R_i) \tag{7}$$

The term $(1 - R_i)$ is known as severity or Loss Given Default **(LGD)**, already used in regulatory capital.

Vasicek model for default rates

In the **Vasicek model**, the dependence structure among counterparties in the portfolio is simplified by the introduction of a common factor that affects all counterparties.
Thus, it is assumed that the standardized asset log-return X_i of obligor i can be decomposed into:

- a **systematic factor** Y (related to the economy),
- an **idiosyncratic factor** ϵ_i,

such that:

$$X_i = \sqrt{\rho} \cdot Y + \sqrt{1-\rho} \cdot \epsilon_i, \tag{8}$$

where

- Y and all ϵ_i are independent standard normal random variables
- ρ is the (average) asset correlation of the portfolio

It is important to note that, conditional on the realisation of the systematic factor Y, the asset values and the defaults are independent.

Vasicek model for default rates

Under this approach, the counterparty ability to pay is represented by a random variable with distribution $N(0,1)$. The default of a counterparty $(D_i = 1)$ takes place when its capacity to pay falls below the threshold K_i (level of debt), so that:

$$PD_i = P(X_i < K_i) \tag{9}$$

and, if we assume that all obligors have the same default threshold $K_i = K$, then

$$K = \Phi^{-1}(PD_i) \tag{10}$$

Conditioning this expression to the value of the systematic factor $Y = y$ and clearing the idiosyncratic factor, the default rate distribution (DR) of a counterparty i is given by:

$$P(D_i = 1|Y = y) = \Phi\left(\frac{1}{\sqrt{1-\rho}}\left(\Phi^{-1}(PD_i) + \sqrt{\rho}\cdot y\right)\right) \tag{11}$$

Vasicek model for default rates

Equation (11) defines a Bernoulli random variable with parameter PD_i. The default is a dichotomous variable with values 0 or 1. The frequency of $D_i = 1$ is given by the parameter PD_i.

If we assume an homogeneous portfolio, that is:

- with n $(n \to \infty)$ borrowers with similar PD and
- the same sensitivity to the common systematic risk factor Y

then the cumulative distribution function of the default rate (DR) of the portfolio converges to

$$P(DR \leq \alpha) = \Phi\left(\frac{\sqrt{1-\rho}\,\Phi^{-1}(\alpha) - \Phi^{-1}(PD)}{\sqrt{\rho}}\right) \tag{12}$$

Intra-portfolio correlation estimation

The asset **correlation** ρ should be interpreted as a parameter that determines the shape of default rate distribution. It provides an indication how volatile the default rate fluctuates over time.

One technique to **estimate the correlation parameter** ρ is the **method of moments**, which uses the first (mean) and the second (variance) moments of the default rate. The theoretical mean and variance of the Vasicek distribution (Eq. (12)) are given by

$$E[DR] = PD, \tag{13}$$

$$Var(DR) = \Phi_2(\Phi^{-1}(PD), \Phi^{-1}(PD), \rho) - PD^2, \tag{14}$$

where Φ_2 is a bivariate normal distribution.
Given default rate data of credit portfolio one can create homogeneous groups and calculate the sample mean $\hat{\mu}^2$ and sample variance $\hat{s}^2$ as estimates. The method of moments obtains the estimate by solving for $\hat{\rho}$ the equation

$$\hat{\mu}^2 + \hat{s}^2 = \Phi_2(\Phi^{-1}(PD), \Phi^{-1}(PD), \hat{\rho}) \tag{15}$$

Stochastic LGD

Vasicek's model can be extended by assuming that the **LGD is stochastic** rather than deterministic.

A similar approach that the one used to compute the probability of default can be carried out, although it would be necessary to use the historical data of recoveries, which tends to be not deep enough.

The most widespread alternative for modeling random severity is the choice of the **beta distribution**. In this case, the historical data are also required as the mean of the beta distribution. The article

Farinelli, S. and Shkolnikov, M.(2012). Two models of stochastic loss given default. The Journal of Credit Risk, 8 (2), pp. 3–20.

provides two models for stochastic LGD, that are going to be the basis in the proposed economic capital estimation.

Correlation between portfolios

EXTRA:

One of the most important effects that can be considered in economic capital and that is not addressed in regulatory capital is diversification between portfolios, sectors, etc.

In order to estimate this correlation, historical series of macroeconomic variables are usually used, such as the GDP of countries or sectors.

The corresponding correlation matrix must meet certain characteristics:

- Symmetric
- Semi positive definite

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Proposed problem for the study week

Estimate and compare regulatory and economic capital, following the steps above:

- Calculate the regulatory capital using standard and IRB approach.
- Calculate the economic capital, using a simulation model based on the Vasicek approach, which is consistent with the Basel II methodology for the IRB regulatory capital:
  - Estimate the default rate distribution using Vasicek (analytical and simulation). In this step, it is necessary to compute the portfolio correlation ρ. Exposure is given and considered as certain.
  - Estimate an stochastic LGD using the two methods described in Farinelli, S. and Shkolnikov, M.(2012).
  - Study how affects the possible correlation between default and recovery to the estimation of economic capital.

Extra Study how affects the possible correlation between portfolios.

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